# Symmetry Protected Invariant Scattering Properties for Incident Plane Waves of Arbitrary Polarizations

*Qingdong Yang, Weijin Chen, Yuntian Chen,* and Wei Liu**

**Polarization independent Mie scattering of building blocks is fundamental for constructions of optical systems with robust functionalities. Conventional studies for such polarization independence are generally restricted to special states of either linear or circular polarizations, widely neglecting elliptically-polarized states that are generically present in realistic applications. Here, a comprehensive recipe to achieve invariant scattering properties in terms of cross sections for plane waves of arbitrary polarizations is presented, requiring only rotation symmetry and absence of optical activities. It is discovered that sole rotation symmetries can effectively decouple the two scattering channels that originate from the incident circularly polarized plane waves of opposite handedness, leading to invariance of all scattering properties for any polarizations on the same latitude circle of the Poincaré sphere. Further incorporations of extra inversion or mirror symmetries would eliminate the optical activities and thus ensure scattering property invariance for arbitrary polarizations. The all-polarization invariance revealed is induced by the joint functions of discrete spatial symmetries, reciprocity, parity conservation and helicity preservation along the forward direction. This symmetry-protected intrinsic invariance, independent of wavelength or material parameters, is robust against any symmetry-preserving perturbations, which may render extra flexibilities for designing optical devices with stable functionalities.**

## 1. Introduction

Photonic devices that can function robustly for some practical applications require polarization independent responses which are immune to perturbations that can easily perturb one polarization state to another.[1,2] For composite devices such as those consisting of periodic, quasi-periodic or disordered photonic structures,[3,4] usually this can be reduced to an elementary Mie scattering problem:[5] its fundamental building atom needs to exhibit invariant scattering properties for different polarizations.[6–9] To get rid of the polarization dependence actually constitutes a rather seminal problem in Mie theory, for which discrete spatial symmetries[6,10–16] and/or electromagnetic duality symmetry[12,13,17–21] can be employed to secure the scattering invariance.

A common limitation widely shared by previous studies is that the polarization independence obtained covers only some specific polarization states (generally circular or linear polarizations), occupying a rather small proportion of the whole Poincaré sphere that can represent all possible polarizations of plane waves.[1,22] To conduct comprehensive investigations into all possible polarization states for realistic practical applications, restricting to some special polarizations is not sufficient considering the following twofold reasons: i) Scattering invariance for some polarizations does not ensure the invariance for all polarizations throughout the whole Poincaré sphere. For example, even if the scattering properties are fully independent of linear polarizations with arbitrary orientations, the scattering variance could still emerge for states that are elliptically polarized. ii) In realistic photonic devices, those widely explored circularly or linearly polarized states are not really absolute stable, which can be easily converted, by inevitable structural defects or external perturbations, into more generic elliptically polarized states.

In this study we show, by geometric reasoning, how to obtain symmetry-protected invariant scattering properties for plane waves of arbitrary polarizations, relying solely on rotation symmetry and optical activity elimination [identical responses for left- and right-handed circularly polarized (LCP and RCP) incident plane waves]. It is discovered that for a scattering configuration of more than twofold rotation symmetry, the two scattering channels from the LCP and RCP waves are actually decoupled: there is effectively no contribution from their cross interferences for scattering properties including extinction, absorption or total scattering. Based on this discovery, we further

Q. Yang, Dr. W. Chen, Prof. Y. Chen
School of Optical and Electronic Information
Huazhong University of Science and Technology
Wuhan, Hubei 430074, P. R. China
E-mail: yuntian@hust.edu.cn

Prof. Y. Chen
Wuhan National Laboratory for Optoelectronics
Huazhong University of Science and Technology
Wuhan, Hubei 430074, P. R. China

Prof. W. Liu
College for Advanced Interdisciplinary Studies
National University of Defense Technology
Changsha 410073, P. R. China
E-mail: wei.liu.pku@gmail.com

**DOI: 10.1002/lpor.202000496**

reveal subtle connections between scattering invariance and a hierarchy of discrete spatial symmetries: i) Rotation symmetries ($n$-fold, $n \geq 3$) result in invariance of all scattering properties for polarizations on the same latitude circle of the Poincaré sphere; ii) Combined rotation-mirror (perpendicular to the rotation axis) or rotation-inversion symmetries lead to invariant extinctions for arbitrary polarizations, while when Ohmic losses of materials are present, the scattering (and consequently also absorption) could still be variable; iii) Combined rotation-mirror (parallel to the rotation axis) symmetry ensures invariant extinction, scattering and absorption for all polarizations covering the whole Poincaré sphere. Protected by those apparent spatial symmetries and the functioning laws of reciprocity, parity conservation and helicity preservation along the forward direction, the scattering invariance obtained is intrinsic and robust against any non-symmetry-breaking perturbations, which can potentially enrich the toolbox of optical device designs and render extra freedom for more flexible manipulations of light-matter interactions.

## 2. Helicity Preservation for Circular Polarizations along the Forward Direction with Configurations that Exhibit *n*-Fold Rotation ($C_n$, $n \geq 3$) Symmetry

In this work, our discussions of Mie scattering with incident plane waves are based on the circular basis **L** and **R**, which correspond to LCP and RCP light, respectively. When circularly polarized (CP) waves are incident on a structure with $C_n$ symmetry (incident direction $\mathbf{k}$ is parallel to the rotation axis $l$: $\mathbf{k} \| l$), the helicity is preserved along the forward direction: the forward scattered waves are not only CP but also of the same handedness as that of the incident plane waves. This has already been rigorously proved by ref. [12]. In this section, we provide a complementary while more geometric formula-free proof that can directly confirm such helicity preservation, without sacrificing any rigor. We emphasize here that throughout this study, our arguments are restricted to threefold rotation symmetry, which can be quite directly generalized to cover all scenarios of $n \geq 3$.

As a first step, we turn to a seemingly unrelated problem sketched in **Figure 1**a,b: with three identical point charges located on the vertexes of an equilateral triangle, what is the electrostatic force on an extra point charge at the triangle center? Through algebraic calculations based on the Coulomb law, we can get the answer that the force is zero. At the same time, we can reach the same conclusion through pure geometric considerations, without any detailed algebraic manipulations: i) Assume that there is a force on the central charge as indicated by a red vector in Figure 1a; ii) Make a $2\pi/3$ rotation operation on the whole configuration, ending up with what is shown in Figure 1b: both the force and the charges are rotated accordingly; iii) Charge distributions in Figure 1a,b are identical due to the overall symmetry, requiring that the force in Figure 1b (dashed red vector) is the same as that in Figure 1a; iv) The two forces (solid and dashed red vectors) in Figure 1b contradict each other, unless the force is zero. This concludes our proof by contradiction.

Now we turn back to our Mie scattering problem and the scattering configuration (exhibiting $C_3$ symmetry) within a spherical coordinate system is schematically shown in Figure 1c. According to Mie theory,[5] what is observed in the forward direction is

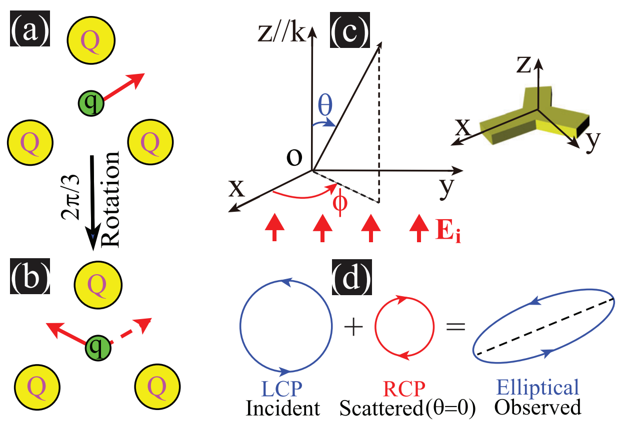


**Figure 1.** a,b) The geometric proof by contradiction that the electrostatic force on the central charge has to be zero with three identical charges located on the vertexes of an equilateral triangle. c) A scattering configuration with $C_3$ symmetry placed in the spherical coordinate system parameterized by polar angle $\theta$ and azimuthal angle $\phi$. d) The geometric proof by contradiction for helicity preservation along the forward direction: for example, a LCP incident plane wave mixing with the forward scattered RCP wave would make the final observable state of elliptical polarization with preferred ellipse orientation, which is forbidden by the rotation symmetry.

mixed states of both incident and forward scattered waves. For incident CP plane waves, let us assume that the helicity is not preserved and thus there is CP scattered components of opposite handedness along the forward direction. It is known that mixing CP waves of opposite handedness would produce elliptically polarized states with preferred orientation directions of the polarization ellipses (in terms of its semi-major or semi-minor axis): a detailed example is shown in Figure 1d with an incident LCP wave mixing with the forward scattered RCP wave. According to the same arguments presented above for Figure 1a,b, such a preferred orientation contradicts the overall rotation symmetry of the scattering configuration with incident CP waves. When there is helicity preservation, the forward mixed state is still CP, for which there is no such contradiction as its ellipse orientation is not defined.[23] In principle, fully flipped helicity in the forward direction is also consistent with the above symmetry arguments. Nevertheless, from a physical perspective, according to the optical theorem,[5] such helicity flipping requires infinite forward scattering (of the same handedness as that of the incident wave) to fully eliminate the incident plane waves through destructive interference. This is not possible for a finite scattering body. This concludes our proof by contradiction for helicity preservation of incident circularly polarized plane waves along the forward direction.

We note that the same symmetry arguments can be also employed to verify the helicity flipping along the backward direction,[12] which nevertheless is irrelevant to our following investigations and thus would not be further discussed in detail here. Moreover, there are actually subtle differences between those shown in Figure 1a,b and Figure 1c,d: the electrostatic force is characterized by a vector that is variant upon the a $\pi$ rotation, and thus $C_2$ symmetry is sufficient to eliminate it; while the polarization ellipse orientation is characterized by a line that is invariant upon a $\pi$ rotation, which means that more than twofold rotation symmetry is required to guarantee the helicity preservation.

 

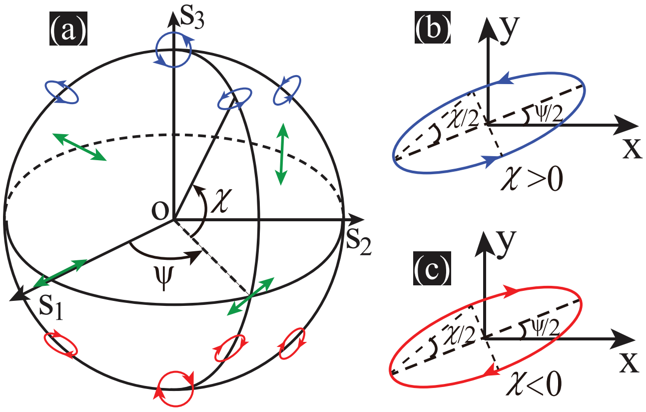


**Figure 2.** a) Representations of arbitrary polarizations by the Poincaré sphere parameterized by Stokes parameters $S_{1,2,3}$, latitude $\chi$ and longitude $\psi$. b,c) The corresponding polarization ellipses are characterized by half angles of those for the Stokes vectors: $\psi/2$ indicates the ellipse orientation and $\chi/2$ describes ellipse eccentricity; polarizations of left (right) handedness locate on the northern (southern) hemisphere with $\chi > 0$ ($\chi < 0$); linear polarizations locate on the equator with $\chi = 0$.

## 3. Invariant Scattering Properties for Arbitrary Polarizations Induced by Discrete Spatial Symmetries

### 3.1. General Theoretical Analysis with Geometric Arguments

In this study we aim to reveal scattering invariance for arbitrary polarizations, and we employ the widely adopted Poincaré sphere to describe polarizations [characterized by Stokes vectors $(S_1, S_2, S_3)$, or location vector $(\chi, \psi)$ in terms of latitude and longitude on a unit-sphere] as shown in **Figure 2**a:[1,22] latitude $\chi \in [-\pi/2, \pi/2]$ characterizes the eccentricities of the polarization ellipses, with positive and negative $\chi$ (northern and southern hemisphere) corresponding to left and right handedness, respectively [see Figure 2b,c]; LCP and RCP waves locate respectively on the northern and southern poles, and all linear polarizations locate on the equator $S_3 = \chi = 0$; latitude $\psi \in [0, 2\pi]$ characterizes the orientations of the polarization ellipses in terms of the semi-major axis (see Figure 2b,c). We emphasize here that the characterizing angles for the polarization ellipses are half of those for Stokes vectors, since the polarizations are described by 1-spinors, which are effectively the square roots of Stokes vectors.[24,25]

An arbitrarily polarized incident plane wave [denoted by $\mathbf{E}_i$; located at $(\chi_i, \psi_i)$ on the Poincaré sphere] can be expressed in circular basis $(\mathbf{L}, \mathbf{R})$ as:

$$\mathbf{E}_\mathrm{i} = \cos\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) \mathrm{e}^{-\mathrm{i}\psi_i/2}\mathbf{L} + \sin\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) \mathrm{e}^{\mathrm{i}\psi_i/2}\mathbf{R}. \tag{1}$$

The scattered waves in the far field (denoted by $\mathbf{E}_\mathrm{s}$) along different directions [characterized by $(\theta, \phi)$ as shown in Figure 1b] are linearly related to the incident plane waves through the following relation:[5]

$$\mathbf{E}_\mathrm{s}(\theta, \phi) = \hat{\mathbf{S}}(\theta, \phi)\mathbf{E}_\mathrm{i} = \cos\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) \mathrm{e}^{-\mathrm{i}\psi_i/2}\mathbf{E}_\mathrm{s}^\mathrm{L}(\theta, \phi) + \sin\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) \mathrm{e}^{\mathrm{i}\psi_i/2}\mathbf{E}_\mathrm{s}^\mathrm{R}(\theta, \phi), \tag{2}$$

where $\hat{\mathbf{S}}$ is the scattering matrix; $\mathbf{E}_\mathrm{s}^\mathrm{L}$ and $\mathbf{E}_\mathrm{s}^\mathrm{R}$ are scattered waves with incident LCP and RCP plane waves, respectively.

For scattering configurations with more than twofold rotation symmetry [the scattering structure exhibits $\mathrm{C}_n$ $(n \geq 3)$ symmetry with the incident plane wave propagating along the rotation axis, as is the case throughout this work], the helicity preservation along the forward direction requires that $\mathbf{E}_\mathrm{s}^\mathrm{L}(\theta = 0)$ and $\mathbf{E}_\mathrm{s}^\mathrm{R}(\theta = 0)$ are respectively LCP and RCP waves, between which there is thus no interference. According to the optical theorem,[5] the extinction is only related to interferences between incident and scattered waves along the forward direction [$\mathbf{E}_\mathrm{i}$ and $\mathbf{E}_\mathrm{s}(\theta = 0)$]. Meanwhile, due to the helicity preservation, there is no cross interference between $\mathbf{L}$ and $\mathbf{E}_\mathrm{s}^\mathrm{R}(\theta = 0)$, or between $\mathbf{R}$ and $\mathbf{E}_\mathrm{s}^\mathrm{L}(\theta = 0)$. As a result, the extinction cross section for arbitrarily polarized incident plane waves can be expressed as:

$$C_\mathrm{ext} = \cos^2\left(\frac{\pi}{4} - \frac{\chi_\mathrm{i}}{2}\right) C_\mathrm{ext}^\mathrm{L} + \sin^2\left(\frac{\pi}{4} - \frac{\chi_\mathrm{i}}{2}\right) C_\mathrm{ext}^\mathrm{R}, \tag{3}$$

where $C_\mathrm{ext}^\mathrm{L}$ and $C_\mathrm{ext}^\mathrm{R}$ are extinction cross sections for incident LCP and RCP plane waves, respectively. According to Equation (3), the extinction has nothing to do with $\psi_i$ (the orientation of incident polarization ellipse) and is invariant: i) for the incident polarizations that locate on the same circle of latitude ($\chi_\mathrm{i}$ is constant); or ii) for arbitrary incident polarizations when there is no extinction activity $\mathrm{C}_\mathrm{ext}^\mathrm{L} = \mathrm{C}_\mathrm{ext}^\mathrm{R}$.[16]

Then we turn to scattering cross sections ($C_\mathrm{sca}$) for any incident polarizations, the calculation of which appears to be more demanding than extinction since integrations of all-angular scattering intensities [$I_\mathrm{s}(\theta, \phi) = |\mathbf{E}_\mathrm{s}(\theta, \phi)|^2$] have to be implemented.[5] According to Equation (2), the angular scattering intensity can be explicitly expressed as:

$$I_\mathrm{s}(\theta, \phi) = \cos^2\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) I_\mathrm{s}^\mathrm{L} + \sin^2\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) I_\mathrm{s}^\mathrm{R} + \cos\left(\chi_i\right) \mathrm{Re}\left(\mathrm{e}^{\mathrm{i}\psi_i}\mathbf{E}_\mathrm{s}^{*\mathrm{L}} \cdot \mathbf{E}_\mathrm{s}^\mathrm{R}\right), \tag{4}$$

where $*$ means complex conjugation. Though along the forward direction $\mathbf{E}_\mathrm{s}^{*\mathrm{L}}$ and $\mathbf{E}_\mathrm{s}^\mathrm{R}$ are orthogonal as required by helicity preservation ($\mathbf{E}_\mathrm{s}^{*\mathrm{L}} \cdot \mathbf{E}_\mathrm{s}^\mathrm{R} = 0$ when $\theta = 0$), they are generally not orthogonal along other directions and thus the last inference term $\left(I_s^\mathrm{int} = \mathrm{Re}\left(\mathrm{e}^{\mathrm{i}\psi_i}\mathbf{E}_\mathrm{s}^{*\mathrm{L}} \cdot \mathbf{E}_\mathrm{s}^\mathrm{R}\right)\right)$ cannot be directly dismissed. This seemingly further adds to the complexities for general discussions of scattering cross sections.

Now we proceed to check in detail the scattered field distributions in terms of both phase and amplitude. For incident CP plane waves, the symmetry ($\mathrm{C}_3$ for the following specific discussions) of the scattering configuration ensures that $\mathbf{E}_\mathrm{s}^{\mathrm{L,\,R}}(\theta, \phi)$ and $\mathbf{E}_\mathrm{s}^{\mathrm{L,\,R}}(\theta, \phi + 2m\pi/3)$ are interconnected rather than independent for $m = 1, 2$. It could be taken for granted that $\mathbf{E}_\mathrm{s}^{\mathrm{L,\,R}}(\theta, \phi) = \mathbf{E}_\mathrm{s}^{\mathrm{L,\,R}}(\theta, \phi + 2m\pi/3)$, as they are seemingly equivalent and interconvertible through a simple coordinate system rotation by $2m\pi/3$ along the $z$-axis (see Figure 1c). This is actually wrong, because such a coordinate rotation does not really leave the scattering configuration as it was, but would rather ultimately change it through introducing extra phase. To be specific, with the coordinate rotation of $2m\pi/3$ along the $z$-axis, the incident CP plane waves would transform as follows (refer to the Feynman Lectures

 

(Volume III, Chapter 11) for more details[26] and the same transformations are also discussed in Ref. [12]:

$$\mathbb{L} = e^{-2m\pi i/3}\mathbf{L}, \quad \mathbb{R} = e^{2m\pi i/3}\mathbf{R}, \tag{5}$$

where the parameters $\mathbb{L}$ and $\mathbb{R}$ (also $\mathbb{E}$, $\mathbb{I}$, and $\hat{\mathbb{S}}$ in the following discussions) are correspondingly those in the new coordinate system after rotation. The scattered fields would also transform accordingly:

$$\mathbb{E}_s^{\mathbb{L}}(\theta,\phi) = e^{-2m\pi i/3}\mathbf{E}_s^{\mathrm{L}}(\theta,\phi),\ \mathbb{E}_s^{\mathbb{R}}(\theta,\phi) = e^{2m\pi i/3}\mathbf{E}_s^{\mathrm{R}}(\theta,\phi), \tag{6}$$

since the scattering matrix is invariant for such a coordinate rotation $\hat{\mathbb{S}}(\theta,\phi) = \hat{\mathbf{S}}(\theta,\phi)$, as protected by the $C_3$ symmetry of the scatterer. Equation (6) leads to the following transformation for the interference term of the scattering intensity in Equation (4):

$$\mathbb{I}_s^{\mathrm{int}}(\theta,\phi) = \cos(4m\pi/3) I_s^{\mathrm{int}}(\theta,\phi) - \sin(4m\pi/3)\,\mathrm{Im}\left(e^{i\psi_i}\mathbf{E}_s^{*\mathrm{L}}\cdot\mathbf{E}_s^{\mathrm{R}}\right). \tag{7}$$

Because an azimuthal angle $\phi$ in the new coordinate system would correspond to $\phi + 2m\pi/3$ in the previous one before rotation, we have $\mathbb{I}_s^{\mathrm{int}}(\theta,\phi) = I_s^{\mathrm{int}}(\theta,\phi + 2m\pi/3)$. As a result, the interference term $I_s^{\mathrm{int}}$ exhibits the following spatial distribution pattern:

$$I_s^{\mathrm{int}}(\theta,\phi + 2m\pi/3) = \cos(4m\pi/3) I_s^{\mathrm{int}}(\theta,\phi) - \sin(4m\pi/3)\,\mathrm{Im}\left(e^{i\psi_i}\mathbf{E}_s^{*\mathrm{L}}\cdot\mathbf{E}_s^{\mathrm{R}}\right). \tag{8}$$

Equation (8) confirms that the interference term would be cancelled out when integrated along all scattering directions [the scattering cross sections have nothing to do with scattering intensities along some specific directions but are decided by the integrated scattering intensities (which are effectively integrated energy flows considering the transverse nature of the scattered fields in the far zone) all across the momentum sphere], since $\cos(0) + \cos(4\pi/3) + \cos(8\pi/3) = 0$ and $\sin(0) + \sin(4\pi/3) + \sin(8\pi/3) = 0$. It immediately becomes clear that $C_2$ symmetry does not guarantee the interference cancellation and thus there is no polarization independence, as $\cos(0) + \cos(2\pi) \neq 0$. To be brief, the scattering cross section for arbitrarily polarized incident plane waves can be simplified as (for more than twofold rotation symmetry):

$$C_{\mathrm{sca}} = \cos^2\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) C_{\mathrm{sca}}^{\mathrm{L}} + \sin^2\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) C_{\mathrm{sca}}^{\mathrm{R}}, \tag{9}$$

where $C_{\mathrm{sca}}^{\mathrm{L}}$ and $C_{\mathrm{sca}}^{\mathrm{R}}$ are scattering cross sections for incident LCP and RCP plane waves, respectively. Similar to Equation (3), Equation (9) confirms that the scattering is invariant: i) for the incident polarizations that locate on the same circle of latitude ($\chi_i$ is constant); or ii) for arbitrary incident polarizations when there is no scattering activity $C_{\mathrm{sca}}^{\mathrm{L}} = C_{\mathrm{sca}}^{\mathrm{R}}$.[16] It now becomes clear that the all-angle integration actually simplifies rather than complexifies the expressions of $C_{\mathrm{sca}}$, but for scattering configurations with rotation symmetry only.

Up to now, we have discussed only extinction and scattering, and properties of absorption can be simply deduced as the absorption cross section can be obtained through the following relation $C_{\mathrm{abs}} = C_{\mathrm{ext}} - C_{\mathrm{sca}}$, as secured by the energy conservation. According to Equations (3) and (9), this leads to:

$$C_{\mathrm{abs}} = \cos^2\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) C_{\mathrm{abs}}^{\mathrm{L}} + \sin^2\left(\frac{\pi}{4} - \frac{\chi_i}{2}\right) C_{\mathrm{abs}}^{\mathrm{R}}, \tag{10}$$

where $C_{\mathrm{abs}}^{\mathrm{L}}$ and $C_{\mathrm{abs}}^{\mathrm{R}}$ are absorption cross sections for incident LCP and RCP plane waves, respectively. The principles we have revealed here for $C_3$ symmetry can be directly extended to any rotation symmetry $C_n$ ($n > 3$). We emphasize that the phase term introduced by coordinate system rotation [shown in Equations (5) and (6)] is only observable through the interference term in Equation (4). When it is CP incident plane waves, there is no such interference and then such a phase is not observable, confirming that the arguments we presented proving the helicity preservation are still valid and not affected by the presence of such a phase.

### 3.2. Invariant Scattering Properties Induced by Sole Rotation Symmetries

Equations (3), (9), and (10) are the core results of our study, requiring the only precondition that the scattering configuration is of more than twofold rotation symmetry. They indicate that for extinction, absorption and total scattering, there is no effective contribution from the interferences between the two scattering channels with incident LCP and RCP plane waves, respectively. This is to say, the two scattering channels are effectively decoupled in terms of the scattering properties we discuss in this work. As a result, all scattering properties for arbitrarily polarized incident plane waves can be obtained by direct summations of the contributions from LCP and RCP incidences. It is clear from those equations that all scattering properties are invariant for polarizations on the same circle of latitude ($\chi_i$ is constant), a special case of which is the linear polarization ($\chi_i = 0$) independence discussed in Ref. [11]. Despite that our conclusion in this work is more general (not limited to linear polarizations), the proof we have presented here is superior: the former proof in Ref. [11] relies on the coupled dipole approximation (thus non-intrinsic) and extremely heavy algebraic manipulations (thus less accessible for general interest) that significantly obscure the physical principles; while here we provide an intrinsic symmetry-based proof assisted by geometric reasoning, which brings to the surface the key mechanisms and thus more comprehensible for the broad community in photonics.

To verify what has been claimed above, we show in **Figure 3**a a composite scattering configuration exhibiting sole $C_3$ symmetry: the two touching scatterers are made of gold, permittivity of which is taken from Ref. [27]; the geometric parameters are specified in the figure and numerical results are obtained using COMSOL Multiphysics, as is the case throughout this work. Despite the invariance of all scattering properties (only extinction and absorption spectra are shown with respect to wavelength $\lambda$) for linear polarizations (see Figure 3b), we also demonstrate in Figure 3c such invariance for elliptic polarizations on the same latitude circle of the Poincaré sphere ($\chi_i = 60°$). For polarizations

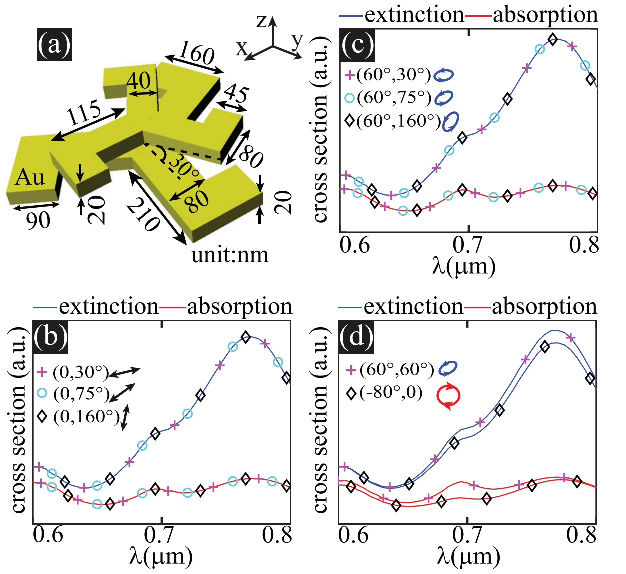


**Figure 3.** a) A scattering configuration consisting of two touching gold particles (geometric parameters specified in the figure only) that exhibits $C_3$ symmetry only. b–d) The extinction and absorption spectra are summarized for: linear polarizations of different orientations in (b); elliptical polarizations of different orientations while on the same lattitude circle of the Poincaré sphere in (c); two randomly chosen polarizations not on the same latitude circle in (d). The labeling angles correspond to the location vector ($\chi_i$, $\psi_i$). The spectra shown in (b) and (c) seem to be identical while actually different in terms of magnitude.

not on the same latitude circle, such invariance is immediately lost, as is shown in Figure 3d.

### 3.3. Invariant Scattering Properties Induced by Combined Rotation-Mirror (Perpendicular to the Rotation Axis) or Rotation-Inversion Symmetries

According to Equations (3), (9), and (10), to obtain invariant extinction, scattering or absorption for arbitrary polarizations (not limited to the same latitude circle on the Poincaré sphere), we only have to extinguish the corresponding scattering activities to ensure equal responses for incident LCP and RCP plane waves:[16] $C^{\mathrm{L}}_{\mathrm{ext,sca,abs}} = C^{\mathrm{R}}_{\mathrm{ext,sca,abs}}$, respectively.

It is recently proved that the law of reciprocity and parity conservation can intrinsically eliminate the extinction activity (but the scattering and absorption activities are still present) when there is mirror (perpendicular to the incident direction) or inversion symmetry.[16] Two such scattering configurations are shown in **Figure 4**a,c, which besides rotation symmetry also exhibit perpendicular mirror and inversion symmetry, respectively. The scattering spectra (in terms of extinction and absorption) are shown respectively in Figure 4b,d for two randomly chosen polarizations (not on the same latitude circle). As is clearly shown there is extinction invariance but no such invariance for absorption or scattering (the two absorption spectra in Figure 4d are quite close, but definitely different, which is most visible close to the wavelength $\lambda = 0.6$ μm), since the scattering and absorption activities are not eliminated by the extra mirror or inversion symmetry.[16]

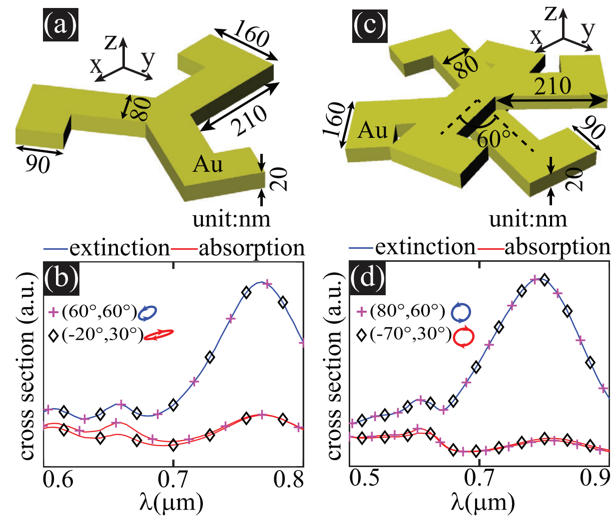


**Figure 4.** Two $C_3$-symmetric scattering configurations consisting of gold particles that exhibit a) extra perpendicular-mirror symmetry and c) inversion symmetry with two identical particles. The b) extinction and d) absorption spectra are summarized respectively, for two randomly chosen polarizations not on the same latitude circle of the Poincaré sphere. The labeling angles correspond to the location vector ($\chi_i$, $\psi_i$).

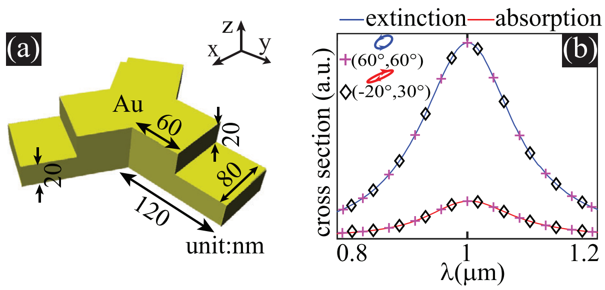


**Figure 5.** a) A scattering configuration consisting of two touching gold particles that exhibit both rotation and parallel-mirror ($C_{3v}$) symmetry. b) The extinction and absorption spectra are shown for two randomly chosen polarizations not on the same latitude circle. The labeling angles correspond to the location vector ($\chi_i$, $\psi_i$).

### 3.4. Invariant Scattering Properties Induced by Combined Rotation-Mirror (Parallel to the Rotation Axis) Symmetries

Equations (3), (9), and (10) tell that to achieve invariance for all scattering properties, all the scattering activities should be eliminated simultaneously. This is possible for a configuration with extra mirror (parallel to the incident direction) symmetry, where the law of parity conservation ensures identical responses for incident LCP and RCP plane waves.[6,16] A scattering configuration exhibiting both rotation and parallel mirror symmetry (with broken perpendicular mirror symmetry) is shown in **Figure 5**a, with the scattering and absorption spectra for two arbitrary polarizations shown in Figure 5b, confirming the invariance of all scattering properties.

## 4. Conclusions and Perspectives

In conclusion, we have revealed through geometric reasoning, how to achieve invariant scattering properties in terms of cross sections for arbitrary polarizations based on discrete spatial symmetries. It is discovered that sole rotational symmetries (more than twofold) can secure the scattering invariance for polarizations on the same latitude circle on the Poincaré sphere. To achieve invariance for all polarizations covering the whole Poincaré sphere, besides rotation symmetry we can: i) introduce extra perpendicular mirror symmetry or inversion symmetry to produce all-polarization invariant extinctions (but not scattering or absorption); ii) or introduce extra parallel mirror symmetry to obtain all-polarization invariant extinction, scattering and absorption. Besides those apparent spatial symmetries, there are also functioning laws of reciprocity, helicity, and parity conservation, which guarantee that the invariance obtained is intrinsic and robust against any symmetry-preserving structure defects or perturbations.

Here in this study, we have confined our discussions to fully-polarized incident plane waves on the Poincaré sphere, but neglect those unpolarized and partially polarized states that are within the Poincaré sphere. Since our core results shown in Equations (3), (9), and (10) have nothing to do with the cross interference term or relative phase difference between two different scattering channels, and thus all equations are valid for states within the Poincaré sphere. It is worth noting that for unpolarized incident light, the interference term is automatically cancelled and thus the validity of Equations (3), (9), and (10) does not reside on the rotation symmetry of the scattering configuration anymore. It is revealed that to obtain invariance for arbitrary polarizations, the absence of optical activities is crucial, which can be either intrinsic (protected by symmetry and thus is broadband as shown in this work) or accidental (activities are only eliminated at some specific wavelengths).

The conclusions we have drawn in this study are restricted to incident plane waves only, and do not generally hold for other incidences of more sophisticated structured beams.[28–31] Nevertheless, since structured beams can be expanded into a series of plane waves, it is reasonable to expect that the framework we have established here might serve as a concrete platform for further discussions of more complicated scattering invariance problems. Moreover, the scattering invariance we have obtained is in the far field and cannot be simply extended to the near field. For example, when linearly polarized plane waves shine on a sphere which exhibits more symmetries than required by our study, the near fields are not invariant and certainly dependent on the orientations of the incident linear polarizations. The principles we have revealed in this work and also the novel approaches we have employed in our geometric reasoning can shed new light on not only optical device designs, but also on fundamental explorations in photonics where the light-matter interactions are dictated by symmetry.

## Acknowledgements

Q.Y. and W.C. contributed equally to this work. The authors acknowledge the financial support from National Natural Science Foundation of China (Grant No. 11874026, 11404403, and 11874426), and the Outstanding Young Researcher Scheme of National University of Defense Technology.

## Conflict of Interest

The authors declare no conflict of interest.

## Data Availability Statement

Research data are not shared.

## Keywords

light polarization, Mie scattering, Poincaré sphere, rotation symmetry, scattering invariance

Received: November 9, 2020
Revised: February 12, 2021
Published online: April 14, 2021

[1] A. Yariv, P. Yeh, *Photonics: Optical Electronics in Modern Communications*, 6th ed., Oxford University Press, New York **2006**.
[2] J.-M. Liu, *Photonic Devices*, Cambridge University Press, Cambridge; New York **2005**.
[3] Z. V. Vardeny, A. Nahata, A. Agrawal, *Nat. Photonics* **2013**, *7*, 177.
[4] D. S. Wiersma, *Nat. Photonics* **2013**, *7*, 188.
[5] C. F. Bohren, D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, Wiley, New York **1983**.
[6] L. D. Barron, *Molecular Light Scattering and Optical Activity*, Cambridge University Press, Cambridge **2009**.
[7] S. Jahani, Z. Jacob, *Nat. Nanotechnol.* **2016**, *11*, 23.
[8] A. I. Kuznetsov, A. E. Miroshnichenko, M. L. Brongersma, Y. S. Kivshar, B. Luk'yanchuk, *Science* **2016**, *354*, aag2472.
[9] W. Liu, Y. S. Kivshar, *Opt. Express* **2018**, *26*, 13085.
[10] R. R. Birss, *Symmetry and Magnetism*, 1st ed., North-Holland Publishing, Amsterdam **1964**.
[11] B. Hopkins, W. Liu, A. E. Miroshnichenko, Y. S. Kivshar, *Nanoscale* **2013**, *5*, 6395.
[12] I. Fernandez-Corbaton, *Opt. Express* **2013**, *21*, 29885.
[13] X. Zambrana-Puyalto, I. Fernandez-Corbaton, M. L. Juan, X. Vidal, G. Molina-Terriza, *Opt. Lett.* **2013**, *38*, 1857.
[14] B. Hopkins, A. N. Poddubny, A. E. Miroshnichenko, Y. S. Kivshar, *Laser Photonics Rev.* **2016**, *10*, 137.
[15] J. T. Collins, C. Kuppe, D. C. Hooper, C. Sibilia, M. Centini, V. K. Valev, *Adv. Opt. Mater.* **2017**, *5*, 1700182.
[16] W. Chen, Q. Yang, Y. Chen, W. Liu, *Phys. Rev. Research* **2020**, *2*, 013277.
[17] I. Fernandez-Corbaton, X. Zambrana-Puyalto, G. Molina-Terriza, *Phys. Rev. A* **2012**, *86*, 042103.
[18] I. Fernandez-Corbaton, X. Zambrana-Puyalto, N. Tischler, X. Vidal, M. L. Juan, G. Molina-Terriza, *Phys. Rev. Lett.* **2013**, *111*, 060401.
[19] A. Rahimzadegan, C. Rockstuhl, I. Fernandez-Corbaton, *Phys. Rev. Applied* **2018**, *9*, 054051.
[20] Q. Yang, W. Chen, Y. Chen, W. Liu, *ACS Photonics* **2020**, *7*, 1830.
[21] Q. Yang, W. Chen, Y. Chen, W. Liu, *Phys. Rev. A* **2020**, *102*, 033517.
[22] S. Ramaseshan, *Curr. Sci.* **1990**, *59*, 1154.
[23] J. F. Nye, *Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations*, CRC Press, Boca Raton, FL **1999**.
[24] E. Cartan, *The Theory of Spinors*, Courier Corporation, NY **2012**.
[25] G. Farmelo, *The Strangest Man: The Hidden Life of Paul Dirac, Mystic of the Atom*, Basic Books, New York **2011**.
[26] R. P. Feynman, R. B. Leighton, M. Sands, *The Feynman Lectures on Physics, Vol. III: Quantum Mechanics*, Basic Books, New York **2011**.
[27] P. B. Johnson, R. W. Christy, *Phys. Rev. B* **1972**, *6*, 4370.
[28] G. Gouesbet, G. Gréhan, *Generalized Lorenz-Mie Theories*, Springer Science & Business Media, Berlin **2011**.
[29] X. Zambrana-Puyalto, X. Vidal, M. L. Juan, G. Molina-Terriza, *Opt. Express* **2013**, *21*, 17520.
[30] X. Zambrana-Puyalto, X. Vidal, G. Molina-Terriza, *Nat. Commun.* **2014**, *5*, 4922.
[31] P. Woźniak, P. Banzer, G. Leuchs, *Laser Photonics Rev.* **2015**, *9*, 231.

 