# Arbitrary polarization-independent backscattering or reflection by rotationally symmetric reciprocal structures

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We study the backward scatterings of plane waves by reciprocal scatterers and reveal that $n$-fold ($n \geqslant 3$) rotation symmetry is sufficient to secure invariant backscattering for arbitrarily polarized incident plane waves. It is further demonstrated that the same principle is also applicable for infinite periodic structures in terms of reflection, which simultaneously guarantees the transmission invariance if there are neither Ohmic losses nor extra diffraction channels. In the presence of losses, extra reflection symmetries (with reflection planes either parallel or perpendicular to the incident direction) can be incorporated to ensure simultaneously the invariance of transmission and absorption. The principles we have revealed are protected by fundamental laws of reciprocity and parity conservation, which are fully independent of the optical or geometric parameters of the photonic structures. The optical invariance obtained is intrinsically robust against perturbations that preserve reciprocity and the geometric symmetries, which could be widely employed for photonic applications that require stable backscatterings or reflections.

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## I. INTRODUCTION

The study of backscatterings (for finite scattering bodies) or reflections (for extended infinite structures that could be homogeneous, periodic, or quasiperiodic) constitutes one of the oldest and most fundamental topics in photonics that spawns many practical applications [1–4]. In the past three decades, largely due to the explosive developments of photonic crystals, metamaterials, and several other sibling disciplines, this topic has attracted surging renewed interest [2,3,5]. In particular, the introduction of emerging new concepts from those fields (including optically induced magnetism, generalized Kerker scattering, a bound state in the continuum, optical topology and non-Hermiticity, and parity-time symmetry) has significantly broadened the horizons and rendered unprecedented flexibilities for efficient manipulations for backward scatterings or reflections [6–18]. Despite those great achievements, there has been an almost pervasive limitation for investigations concerning backscattering or reflection: the studies have been largely confined to incident waves of either circular (for chiroptical studies in particular [4]) or linear polarizations. However, for practical applications, arbitrary polarization-independent responses are essential for stable operations against polarization fluctuations induced by perturbations, systematic discussions of which are vitally important but, unfortunately, rare.

Recently, we showed comprehensively how to exploit simultaneously rotation (or electromagnetic duality) and reflection symmetries to deliver invariant scattering properties of reciprocal optical scatters [19,20] for arbitrary polarizations covering the whole Poincaré sphere [2]. Nevertheless, the invariance obtained concerns only the integrated scattering properties of cross sections (including extinction, absorption, and scattering cross sections), and it has not been discussed specifically whether or not scatterings along fixed angles (backward scattering, for example) are invariant or not. Moreover, it is well known that optical responses of periodic photonic structures are dictated by the unit-cell particle scatterings (lattice coupling taken into consideration) along the corresponding out-coupling directions, e.g., forward scatterings for transmissions, backward scatterings for reflections, and scatterings along diffractive directions for diffractions of different orders [21,22]. As a result, discussing the scattering invariance along some fixed directions for arbitrary polarizations is of great significance for both finite scatterers and extended periodic structures.

In this study we discuss the backscatterings of arbitrarily polarized incident plane waves by reciprocal scattering bodies. It is revealed that sole $n$-fold ($n \geqslant 3$) rotation symmetry can ensure invariant backward scattering irrespective of either optical or geometric parameters (rotation axis parallel to the incident direction). Similarly, reflection invariance can be also obtained for rotationally symmetric periodic structures, which

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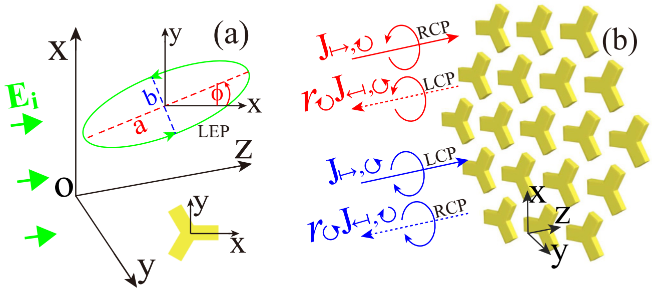


FIG. 1. (a) Plane wave (propagating along the $+\mathbf{z}$ direction) scattering by rotationally symmetric obstacles (symmetry axis parallel to the $\mathbf{z}$ axis). For generally elliptically polarized incident waves, the polarization ellipse can be characterized by eccentricity $e = \pm b/a$ (the inset shows a left-handed ellipse with $e > 0$) and orientation angle $\phi$. (b) Schematic illustration of the backscattering or reflection processes for incident circularly polarized waves. The backscattered or reflected waves are also circularly polarized with opposite handedness.

is accompanied by invariant transmission when there are neither Ohmic losses nor extra diffractions. In the presence of losses, it is further demonstrated that extra reflection symmetries (reflection planes either parallel or perpendicular to the incident direction) can be introduced to produce invariant absorptions and transmissions. The invariant responses obtained are generically protected by reciprocity and geometric symmetries, which may help to construct practical photonic devices with robust optical functionalities. This work is organized as follows: detailed theoretical analysis based on reciprocity and rotation symmetry arguments is presented in Sec. II, which are then verified by numerical studies of finite scattering bodies (Sec. III) and infinite periodic structures (Sec. IV). In Sec. V clusters of self-dual particles are discussed, and it is shown backscattering or reflection invariance is not protected by self-duality of scattering bodies; the paper is concluded in Sec. VI.

## II. THEORETICAL ANALYSIS BASED ON RECIPROCITY AND ROTATION SYMMETRIES

The scattering configuration we study is shown schematically in Fig. 1(a): the incident plane wave is propagating along the $+\mathbf{z}$ direction, and the $n$-fold ($n \geqslant 3$) rotation symmetry axis of the scattering body is also parallel to the $\mathbf{z}$ axis. Here we show only a threefold rotationally symmetric scatterer but note that the principles we reveal are applicable to all scenarios with $n \geqslant 3$. For the analysis in this study, $\mapsto$ and $\mapsfrom$ denote waves propagating along the $+\mathbf{z}$ and $-\mathbf{z}$ directions, respectively; $\circlearrowleft$ and $\circlearrowright$ denote waves of left-handed circularly polarized (LCP) and right-handed circularly polarized (RCP) waves, respectively (viewed counter to the direction of propagation). With the temporal phase component $\exp(-i\omega t)$ dropped, the normalized complex Jones vectors for the four possible combinations are [2]

$$\mathbf{J}_{\mapsto,\circlearrowleft} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}e^{ikz}, \quad \mathbf{J}_{\mapsfrom,\circlearrowleft} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}e^{-ikz},$$
$$\mathbf{J}_{\mapsto,\circlearrowright} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}e^{ikz}, \quad \mathbf{J}_{\mapsfrom,\circlearrowright} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}e^{-ikz}. \quad (1)$$

For circularly polarized incident waves of $\mathbf{J}_{\mapsto,\circlearrowleft}$ and $\mathbf{J}_{\mapsto,\circlearrowright}$, the $n$-fold ($n \geqslant 3$) rotation symmetry of the scattering body ensures that the backscattered waves are also circularly polarized of the opposite handedness [19,23]. The backscattering processes can be expressed as [also shown schematically in Fig. 1(b)]

$$\mathbf{J}_{\mapsto,\circlearrowleft} \Rightarrow r_{\circlearrowleft}\mathbf{J}_{\mapsfrom,\circlearrowright},$$
$$\mathbf{J}_{\mapsto,\circlearrowright} \Rightarrow r_{\circlearrowright}\mathbf{J}_{\mapsfrom,\circlearrowleft}, \quad (2)$$

where $r_{\circlearrowleft}$ and $r_{\circlearrowright}$ denote the backscattering coefficients for incident LCP and RCP waves, respectively. Equation (2) basically shows that upon backscattering, both the propagation direction and the handedness of the wave are flipped.

Similar to the backscattering process for rotationally symmetric scatterers, a time reversal operation $\mathbf{T}$ also reverses the wave propagation direction but maintains the wave handedness:

$$\hat{\mathbf{T}}: \mathbf{J}_{\mapsto,\circlearrowleft} \Leftrightarrow \mathbf{J}_{\mapsfrom,\circlearrowleft}, \quad \mathbf{J}_{\mapsto,\circlearrowright} \Leftrightarrow \mathbf{J}_{\mapsfrom,\circlearrowright}, \quad (3)$$

which in our notations is effectively equivalent to the complex conjugate operation (*) since according to Eq. (1),

$$(\mathbf{J}_{\mapsto,\circlearrowleft})^* = \mathbf{J}_{\mapsfrom,\circlearrowleft}, \quad (\mathbf{J}_{\mapsto,\circlearrowright})^* = \mathbf{J}_{\mapsfrom,\circlearrowright}. \quad (4)$$

The principle of electromagnetic reciprocity [24] requires that the incident and backscattered circularly polarized field components have to satisfy the following equation:

$$(\hat{\mathbf{T}}\mathbf{J}_{\mapsto,\circlearrowleft})^\dagger r_{\circlearrowright}\mathbf{J}_{\mapsfrom,\circlearrowleft} = (\hat{\mathbf{T}}\mathbf{J}_{\mapsto,\circlearrowright})^\dagger r_{\circlearrowleft}\mathbf{J}_{\mapsfrom,\circlearrowright}, \quad (5)$$

which according to Eqs. (3) and (4) is effectively

$$(\mathbf{J}^*_{\mapsto,\circlearrowleft})^\dagger r_{\circlearrowright}\mathbf{J}_{\mapsfrom,\circlearrowleft} = (\mathbf{J}^*_{\mapsto,\circlearrowright})^\dagger r_{\circlearrowleft}\mathbf{J}_{\mapsfrom,\circlearrowright}, \quad (6)$$

where † corresponds to the complex-conjugate-transpose operation: $\binom{\alpha}{\beta}^\dagger = (\alpha^*, \beta^*)$ ($\alpha$ and $\beta$ are arbitrary complex numbers). It is easy to confirm that Eqs. (5) and (6) can be rewritten in an unconjugated direct dot product from $[\binom{\alpha_1}{\beta_1}\cdot\binom{\alpha_2}{\beta_2} = \alpha_1\alpha_2 + \beta_1\beta_2]$:

$$(\mathbf{J}_{\mapsto,\circlearrowleft})\cdot(r_{\circlearrowright}\mathbf{J}_{\mapsfrom,\circlearrowleft}) = (\mathbf{J}_{\mapsto,\circlearrowright})\cdot(r_{\circlearrowleft}\mathbf{J}_{\mapsfrom,\circlearrowright}), \quad (7)$$

which is exactly the de Hoop form of reciprocity [24,25]. We emphasize that both forms of reciprocity [those shown in Eqs. (5) and (7)] were employed in previous studies and, as clarified above, are exactly equivalent.

According to Eq. (4), Eq. (6) can be reduced to ($r_{\circlearrowleft}$ and $r_{\circlearrowright}$ are complex scalars)

$$r_{\circlearrowright}(\mathbf{J}_{\mapsfrom,\circlearrowleft})^\dagger\mathbf{J}_{\mapsfrom,\circlearrowleft} = r_{\circlearrowleft}(\mathbf{J}_{\mapsfrom,\circlearrowright})^\dagger\mathbf{J}_{\mapsfrom,\circlearrowright}. \quad (8)$$

It is easy to confirm from Eq. (1) that $(\mathbf{J}_{\mapsfrom,\circlearrowleft})^\dagger\mathbf{J}_{\mapsfrom,\circlearrowleft} = (\mathbf{J}_{\mapsfrom,\circlearrowright})^\dagger\mathbf{J}_{\mapsfrom,\circlearrowright} = 1$ (this is effectively the definition for the squared length of the normalized complex Jones vectors); as a result we obtain the circular-polarization-independent backscattering coefficient and efficiency:

$$r_{\circlearrowleft} = r_{\circlearrowright} = r_c, \quad R_{\circlearrowleft} = R_{\circlearrowright} = |r_c|^2. \quad (9)$$

After studying circular polarizations, now we turn to the scenario of arbitrarily polarized incident waves, which can always be expanded into the circular basis as follows (in terms of the electric field):

$$\mathbf{E}_i = c_{\circlearrowright}\mathbf{J}_{\mapsto,\circlearrowright} + c_{\circlearrowleft}\mathbf{J}_{\mapsto,\circlearrowleft}. \tag{10}$$

Here the incident field intensity is normalized: $|c_{\circlearrowright}|^2 + |c_{\circlearrowleft}|^2 = 1$. We further define the circular polarization component ratio $c = c_{\circlearrowleft}/c_{\circlearrowright}$, where, obviously, $c = 0$ and $c = \infty$ correspond to LCP and RCP waves, respectively. For other general elliptical polarizations [refer to the inset in Fig. 1(a), where a left-handed elliptically polarized (LEP) wave is shown], $|c|$ decides the eccentricity $e = \pm b/a$ (the positive and negative signs correspond to left and right handedness, respectively) and thus also the handedness of the polarization ellipse; its phase Arg($c$) decides the orientation angle $\phi$ of the polarization ellipse (the angle between the $\mathbf{x}$ axis and the semimajor axis of the ellipse) [19,20]. For example, $|c| < 1$, $|c| > 1$, and $|c| = 1$ correspond, respectively, to left-handed, right-handed, and linear polarizations. According to Eq. (2), the backscattered field is

$$\mathbf{E}_r = r_c(c_{\circlearrowright}\mathbf{J}_{\leftrightarrow,\circlearrowleft} + c_{\circlearrowleft}\mathbf{J}_{\leftrightarrow,\circlearrowright}). \tag{11}$$

Since the two reflected components $\mathbf{J}_{\leftrightarrow,\circlearrowleft}$ and $\mathbf{J}_{\leftrightarrow,\circlearrowright}$ are orthogonal, the backscattering coefficient and efficiency for arbitrary polarizations can be expressed as

$$r_a = \frac{\mathbf{E}_r}{\mathbf{E}_i} = r_c,$$
$$R_a = \frac{|\mathbf{E}_r|^2}{|\mathbf{E}_i|^2} = |r_c|^2(|c_{\circlearrowright}|^2 + |c_{\circlearrowleft}|^2) = |r_c|^2, \tag{12}$$

which obviously have nothing to do with the polarization state of the incident plane wave.

We note here that our discussions above are fully based on the fundamental principle of reciprocity and geometric rotation symmetry, which can be directly extended from finite scattering bodies to infinite periodic or even quasiperiodic structures. Then $r_a$ and $R_a$ represent, respectively, reflection coefficient and efficiency, which in a similar fashion are also arbitrarily polarization independent. Despite this similarity, there are also obvious differences between finite scattering bodies and infinite periodic structures. Besides the common absorption channel, for the former there is an infinite number of out-coupling channels that correspond to scatterings along all directions on the momentum sphere; however, for the latter, the number of the out-coupling channels is finite, including reflection, transmission, and other possible diffractions limited by the periodicity. Without absorption (no Ohmic losses) or diffractions (periodicity is small enough to make all diffractions evanescent), rotation symmetries would directly lead to the invariance of both reflections and transmissions, as required by the law of energy conservation.

## III. INVARIANT BACKSCATTERING OF FINITE SCATTERERS

To verify our theoretical analysis above, first, we investigate finite scattering bodies, and the numerically calculated results are shown in Fig. 2 (numerical results in this work are obtained with the commercial software COMSOL MULTIPHYSICS). Here we have studied three gold particles (optical parameters taken from Ref. [26]; geometric parameters specified in Fig. 2) that exhibit threefold, fourfold, and sixfold rotation symmetries, respectively [see Figs. 2(a)–2(c); no other geometric symmetries are shown]. The corresponding backscattering efficiencies are shown, respectively, in Figs. 2(d)–2(f), and for each scenario we show three sets of spectra with different incident polarizations: two circular polarizations of opposite handedness (LCP and RCP) and one LEP with $e = 0.5$ and $\phi = 45^\circ$. We emphasize that in this study we have only randomly chosen three representative incident polarizations and the results are the same for other polarizations as well, as is the case throughout our work. Moreover, the backscattering invariance shown here can be also obtained for any $n$-fold ($n \geqslant 3$) rotationally symmetric scatterers, though here we showcase only three scenarios. Those results have confirmed our conclusion that sole rotation symmetries can ensure arbitrary polarization-independent backscattering by reciprocal scattering bodies.

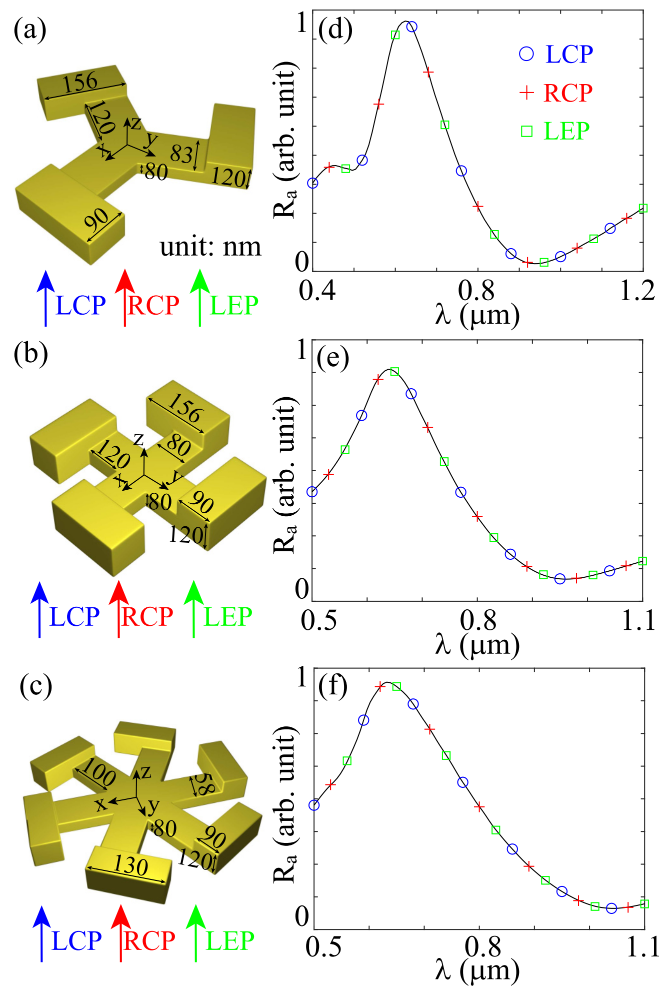


FIG. 2. (a)–(c) Finite scattering bodies made of gold that exhibit threefold, fourfold, and sixfold rotation symmetries, respectively. The geometric parameters are specified. (d)–(f) The corresponding backscattering efficiency spectra for the structures in (a)–(c). For each structure, three sets of spectra are shown for three incident polarizations: LCP, RCP, and LEP ($e = 0.5$ and $\phi = 45^\circ$).

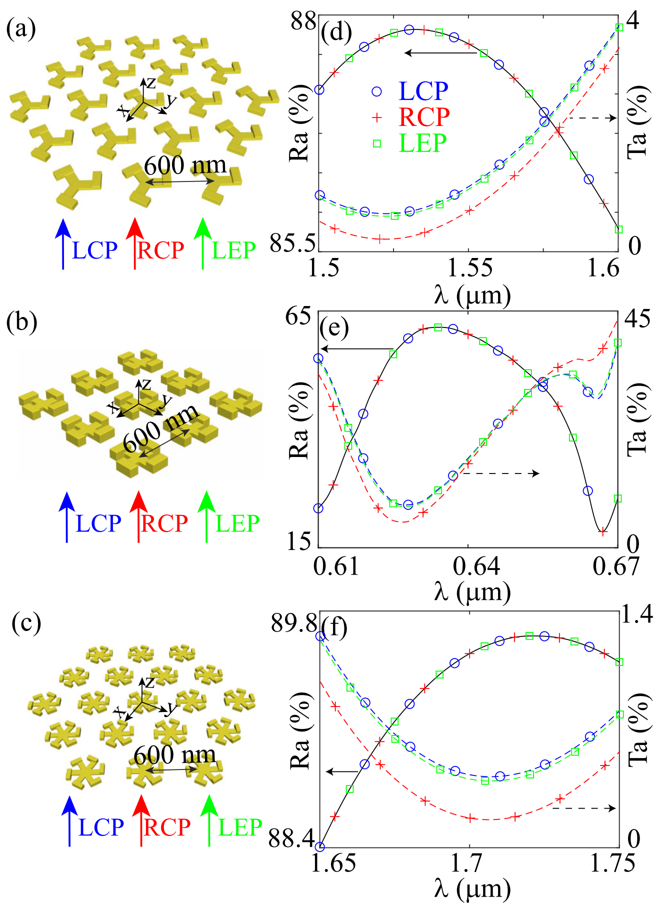


FIG. 3. (a)–(c) Periodic structures [the constituent unit-cell particles are the same as those in Figs. 2(a)–2(c), respectively] that exhibit threefold, fourfold, and sixfold rotation symmetries, respectively. There are two triangular lattices in (a) and (c) and one square lattice in (b). The periodicity parameters are specified. (d)–(f) The corresponding transmission and reflection efficiency spectra for the structures in (a)–(c).

## IV. INVARIANT OPTICAL RESPONSES OF PERIODIC STRUCTURES

### A. Invariant reflections induced by rotation symmetries

As a next step, we turn to extended periodic structures that show threefold, fourfold, and sixfold rotation symmetries [see Figs. 3(a)–3(c), where there are two triangular lattices in Figs. 3(a) and 3(c) and one square lattice in Fig. 3(b)]: the unit-cell particles are the same as those in Figs. 2(a)–2(c), respectively, and the periodicity information is specified in the figure. The corresponding reflection spectra are shown, respectively, in Figs. 3(d)–3(f), where their independence from the polarization is clearly observed. Besides the reflection spectra, we have also included the results for transmission efficiency spectra of arbitrary polarizations $T_a$, for which in contrast there is no such polarization independence. In the spectral regime we study there are no other diffractions, and consequently, this variance of transmission with respect to polarizations must be induced by the Ohmic losses of gold. The law of energy conservation guarantees the coexistence of transmission and reflection invariance when there are neither Ohmic losses nor extra diffractions.

We emphasize here that the reflection invariance for some special scenarios (such as threefold and fourfold rotationally symmetric lattices with circularly polarized incident waves) has already been demonstrated in previous studies [27–30], where relatively more algebraic arguments centered on the scattering matrix were put forward to clarify the mechanisms. In comparison, our arguments in this study are more geometric and intuitive, and most importantly, they are all inclusive, universally covering all $n$-fold ($n \geqslant 3$) rotational symmetries and all polarizations. In addition, our conclusions are valid for both finite scattering bodies and extended periodic (or quasiperiodic) structures, involving no complicated algebraic manipulations of the scattering matrices.

### B. Invariant reflections and transmissions induced by joint rotation-reflection symmetries

Now we proceed to discuss how to obtain both invariance of reflection and transmission in the presence of losses. Our previous studies have shown that $n$-fold ($n \geqslant 3$) rotation symmetries together with extinction invariance for unit-cell scattering bodies can lead to transmission invariance [19,31]. The extinction invariance for arbitrary polarizations can be achieved by introducing extra reflection symmetries, with the reflection plane either perpendicular or parallel to the incident direction (perpendicular or parallel reflection symmetry) [19,31]. The results of the joint rotation-perpendicular (parallel) reflection symmetry scenario are summarized in Fig. 4 (Fig. 5). The periodic structures (two triangular lattices and one square lattice for each scenario; only unit-cell particles are shown) in Figs. 4(a)–4(c) and 5(a)–5(c) are the same as those in Figs. 3(a)–3(c), except now the unit-cell particles are modified to accommodate the corresponding perpendicular or parallel reflection symmetry. The corresponding reflection and transmission spectra in Figs. 4(d)–4(f) and 5(d)–5(f) have verified the invariance of both reflections and transmissions for arbitrary polarizations. When there are no extra diffractions (as is the case for our studies in Fig. 4), it is known from the law of energy conservation that the invariance of reflections and transmissions means also absorption invariance. For the parallel reflection symmetry scenario shown in Fig. 5, the parity conservation and rotation symmetry directly guarantee the invariance of absorption, irrespective of whether there are higher-order diffractions or not [19,31].

We further note that the principles we have revealed concerning the reflection and transmission invariance are secured by the fundamental laws of reciprocity and parity conservation, irrespective of whether or not there are extra diffractions [19,31]. Of course, in the presence of higher-order diffractions, we have to specify that what we mean by reflection and transmission are actually the corresponding zero-order diffractions. At the same time, the invariance of reflection and transmission does not necessarily result in absorption invariance since there are other out-coupling diffraction channels.

## V. ABSENCE OF BACKSCATTERING OR REFLECTION INVARIANCE FOR SELF-DUAL OPTICAL STRUCTURES

As a final step, we turn to self-dual photonic structures that are invariant under the electromagnetic duality

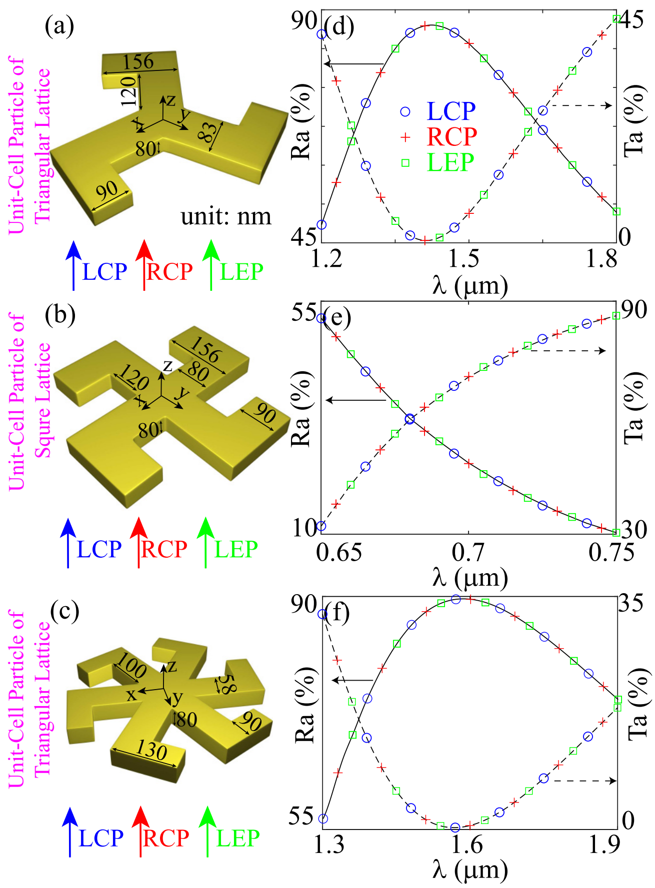


FIG. 4. (a)–(c) Periodic structures [only the unit-cell particles are shown; the periodicity parameters and rotation symmetries are the same as those in Figs. 3(a)–3(c), respectively] that also exhibit extra reflection symmetry with the reflection mirror perpendicular to the incident direction. The geometric parameters of the unit-cell particles are specified. (d)–(f) The corresponding transmission and reflection efficiency spectra for the structures in (a)–(c).

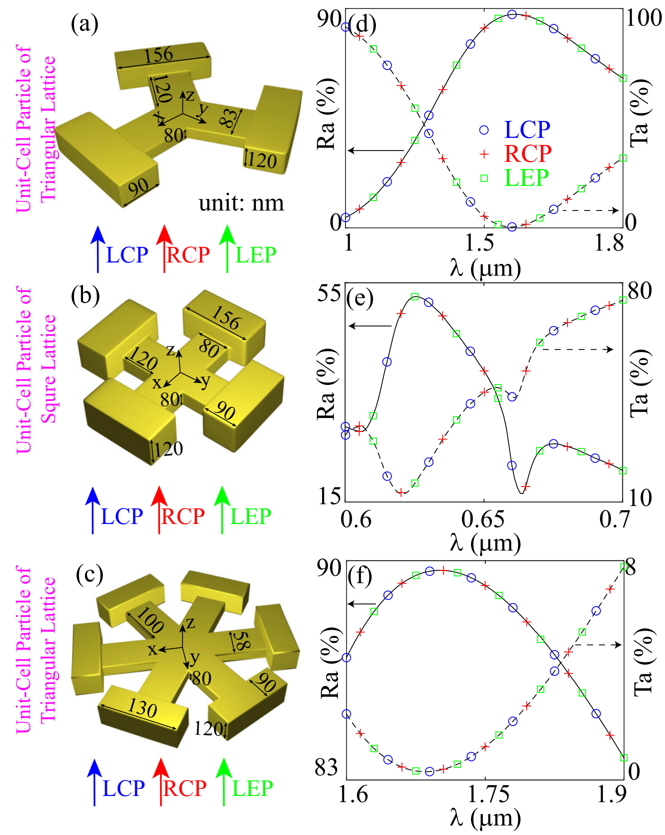


FIG. 5. (a)–(c) Periodic structures [only the unit-cell particles are shown; the periodicity parameters and rotation symmetries are the same as those in Figs. 3(a)–3(c), respectively] that also exhibit extra reflection symmetry with the reflection mirror parallel to the incident direction. The geometric parameters of the unit-cell particles are specified. (d)–(f) The corresponding transmission and reflection efficiency spectra for the structures in (a)–(c).

transformation [20,23,32–34]. This is because self-dual and rotationally symmetric structures are similar in the sense that for incident circularly polarized waves, the backscattered or reflected waves are also circularly polarized (actually, for self-dual ones, along all scattering or out-coupling directions the waves are either circularly polarized or zero) [20,23,33]. The contrasting difference is that the handedness of the waves upon backscattering or reflection is preserved for self-dual structures but flipped for rotationally symmetric ones. As a result, when the formula and analysis in Sec. II are mapped to self-dual structures, Eq. (8) is converted to

$$r_{\circlearrowleft}(\mathbf{J}_{\leftrightarrow,\circlearrowleft})^{\dagger}\mathbf{J}_{\leftrightarrow,\circlearrowright} = r_{\circlearrowright}(\mathbf{J}_{\leftrightarrow,\circlearrowright})^{\dagger}\mathbf{J}_{\leftrightarrow,\circlearrowleft}. \tag{13}$$

According to Eq. (1), $(\mathbf{J}_{\leftrightarrow,\circlearrowleft})^{\dagger}\mathbf{J}_{\leftrightarrow,\circlearrowright} = (\mathbf{J}_{\leftrightarrow,\circlearrowright})^{\dagger}\mathbf{J}_{\leftrightarrow,\circlearrowleft} = 0$ (this is effectively the definition of orthogonality between complex Jones vectors that characterize waves of opposite handedness but the same propagation direction), and consequently, Eq. (13) is generically satisfied, telling nothing about the backscattering coefficients. That is to say, in sharp contrast to the rotationally symmetric scenario, self-duality does not necessarily lead to backscattering even for circular polarizations.

To verify the above analysis, we restudy the self-dual scattering cluster studied in Ref. [20], but now from a different perspective of backscattering efficiency rather than total scattering cross sections. The cluster consists of a Ag core-dielectric shell spherical particle (inner radius of 65 nm and outer radius of 218 nm; the refractive index of Ag is adopted from Ref. [26] and that of the dielectric shell is $n = 3.4$), which is self-dual at the wavelength $\lambda_{\mathbf{SD}} = 1512$ nm (a pair of electric and magnetic dipoles of equal strength is supported at $\lambda_{\mathbf{SD}}$). The scattering configuration is shown schematically in Fig. 6(a), where the cylindrical coordinates ($r = \sqrt{x^2 + y^2}, \phi, z$) of particles are also specified. The backscattering efficiency spectra are summarized in Fig. 6(b), which clearly demonstrates that self-duality at $\lambda_{\mathbf{SD}}$ does not lead to invariance of the backward scattering.

## VI. CONCLUSION

To conclude, we revealed that for reciprocal finite scattering bodies or extended periodic structures, rotation symmetries (no less than threefold) are sufficient to guarantee invariant backscattering or reflection for incident plane waves of arbitrary polarizations. For periodic structures, it was

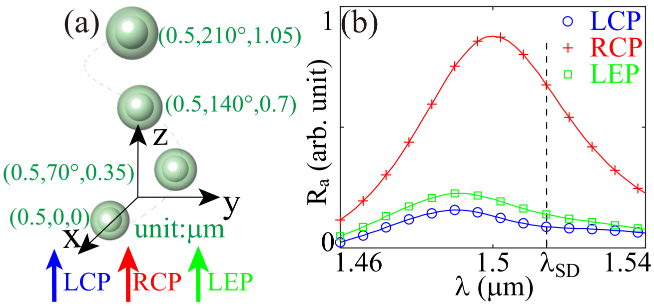


FIG. 6. (a) The scattering configuration consisting of four self-dual (at $\lambda_{SD}$) Ag core-dielectric ($n = 3.4$) shell spherical particles. The cylindrical coordinates of the particles are specified, and this configuration exhibits neither rotation nor reflection symmetry. (b) The corresponding backscattering efficiency spectra for three incident polarizations, with the self-dual wavelength marked by the dashed vertical line.

further shown that the incorporation of extra reflection symmetries (with the reflection plane either perpendicular or parallel to the incident direction) would also secure the polarization independence of transmissions. This means invariant reflections, transmissions, and absorptions when there are no extra higher-order diffraction orders. For the numerical demonstrations in Figs. 3–5, we employed only periodic structures that exhibit threefold, fourfold, and sixfold rotation symmetries, but we emphasize that the conclusions are also valid for quasiperiodic structures that harbor other rotation symmetries. Since the conclusions we have drawn are fully based on fundamental principles of reciprocity, parity conservation, and rotation symmetries, experimental verifications can be conducted in all photonic spectral regimes for arbitrary material compositions and geometric parameters, as long as the structure remains reciprocal and rotationally symmetric. Another advantage of experimental studies is that the effects confirmed theoretically are not affected by the presence of a substrate, which can be easily introduced without breaking the overall structure rotation symmetry. The principles we have revealed may play significant roles in both fundamental explorations involving polarization states of light (such as investigations of chiroptics and the polarization-evolving Pancharatnam-Berry phase [35]) and practical applications that need stable optical functionalities that are robust against polarization fluctuations.

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