Hamiltonian formulation of cross mode modulation

1st Haofan Yang
School of Optical and Electronic Information,
Huazhong University of Science and Technology
Wuhan, China
haofan_yang@hust.edu.cn

2nd Weijin Chen
School of Optical and Electronic Information,
Huazhong University of Science and Technology
Wuhan, China
cwj@hust.edu.cn

3rd Hanwen Hu
School of Optical and Electronic Information,
Huazhong University of Science and Technology
Wuhan, China
hwhu@hust.edu.cn

4th Yuntian Chen
School of Optical and Electronic Information,
Huazhong University of Science and Technology
Wuhan, China
yuntian@hust.edu.cn

5th Jing Xu*
School of Optical and Electronic Information,
Wuhan National Laboratory for Optoelectronics,
Huazhong University of Science and Technology
Wuhan, China
jing_xu@hust.edu.cn

6th Xinliang Zhang
School of Optical and Electronic Information,
Wuhan National Laboratory for Optoelectronics,
Huazhong University of Science and Technology
Wuhan, China
xlzhang@mail.hust.edu.cn

***Abstract*—We investigated the cross mode modulation (XMM) effect of a strong pump light on a weak probe light. The eigenmodes and the corresponding eigenvalues of the nonlinear system can be obtained by using the Hamiltonian approach, i.e., a theoretical framework using a linear coupled mode theory. The evolution of the probe light can be obtained based on eigenmodes and eigenvectors, which agrees with the numerical simulations of the coupled nonlinear Schrödinger equations.**

***Index Terms*—cross mode modulation, Hamiltonian, eigenmodes**

I. Introduction

Cross mode modulation (XMM) is a nonlinear optical process that manifests itself as the variation of the mode profile of a weak probe light when co-propagating with a much stronger pump light [1], which may play a significant role in the spatial-division multiplexing (SDM) transmission systems. From a physical point of view, we can decompose the probe light into a series of eigenmodes which may experience different nonlinear phase shift induced by the pump light. Actually, such pump-induced birefringence is also known as nonlinear polarization rotation (NPR) in the single spatial mode dual polarization configuration [2]. The nonlinear propagation constant induced by the horizontally polarized pump light for the horizontally and vertically polarizations are slightly different, as shown in Fig. 1(a). The evolution of the state of polarization of an input probe light which is linearly polarized at 45° is shown in Fig. 1(b). The situation of XMM is similar. Figure 1(c) and 1(d) shows the case where the pump light is launched in the LP_{11}^b mode. Here LP_{11}^a (LP_{11}^b) refers to the LP_{11} mode with two lobes aligned vertically (horizontally). For an input probe light that is in the LP_{11} mode as well but with two lobes aligned at 45°, it can be equally split into the LP_{11}^a and LP_{11}^b modes which experience different nonlinear propagation constant induced by the pump light. Therefore, the mode field distribution of the probe light is rotated by 90° compared to the incident pattern over one-half of the beat length when the nonlinear phase shift imposed on the LP_{11}^a and LP_{11}^b mode is out of phase. Obviously, the prediction of the mode field distribution of the probe light at different transmission distance that caused by the XMM depends on the concrete form of mode decomposition. In this paper, in such pump-probe configuration, we propose a theoretical framework to obtain the principle axes, i.e., the eigenmodes of the system in terms of the XMM nonlinearity based on the coupled mode theory. Therefore, by calculating the linear superposition of the eigenmodes with the nonlinear phase shifts indicated by the corresponding eigenvalues, we can obtain the mode field distribution of the probe light at different transmission distances. That is to say, by solving the Hamiltonian of the nonlinear system, this complicated nonlinear problem can be greatly simplified into a linear problem.

II. Theoretical Framework

We start from the Dirac notation of the Manakov equation for a mode group of N spatial modes [1]

$$\frac{\partial \left|E\right\rangle}{\partial z} = -\beta_1 \frac{\partial \left|E\right\rangle}{\partial t} - i\frac{\beta_2}{2}\frac{\partial^2 \left|E\right\rangle}{\partial t^2} + i\gamma\kappa \left\langle E|E\right\rangle \left|E\right\rangle . \quad (1)$$

The ket $|E\rangle$ representing the amplitudes of the electric field envelope, $\langle E|$ is the conjugate transpose of $|E\rangle$. β_1 and β_2 represents the group delay per length and dispersion of the group velocity, respectively. κ denotes the random mode

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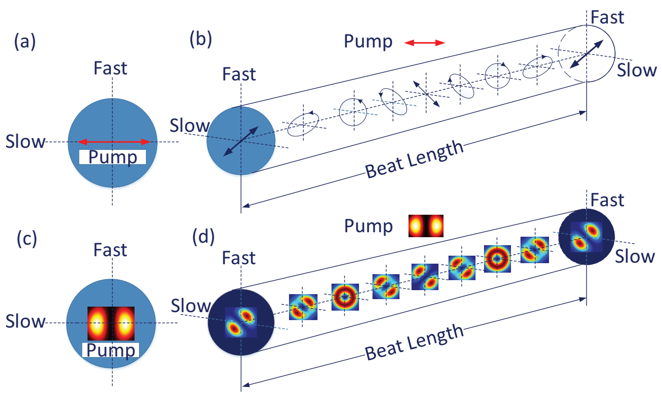

Fig. 1. (a) Nonlinear birefringence introduced by a strong pump light that is launched in the fundamental mode and polarized horizontally. (b) Evolution of the state of polarization of a weak probe light which linear polarized at 45° (from the fast or slow axis) over a beat length. (c) Nonlinear birefringence introduced by a strong pump light that is launched in the LP_{11} mode with two lobes aligned horizontally. (d) Evolution of the mode field distribution of a weak probe light which is launched into the LP_{11} mode with two lobes aligned at 45° over a beat length. The probe light is co-polarized with the pump light in Fig. 1(d).

mixing [3]. The nonlinear parameter $\gamma = \omega_0 n_2 / cA_{eff}$ is the nonlinear parameter defined in the scalar NLSE [4], where n_2 is the nonlinear refractive index, A_{eff} is the effective mode area of the fundamental mode at the angular frequency ω_0, and c is the speed of light. In the pump-probe configuration adopted in this work, the probe signal $|a\rangle$ and the pump $|b\rangle$ is at wavelength λ_a and λ_b, respectively. Thus the total electrical field at the input of the nonlinear waveguide is $|E\rangle = |a\rangle + |b\rangle$. By considering continuous wave (CW) situation for both input light, we can neglect the dispersion effects. The components of $|E\rangle$ thus satisfy the following coupled equations

$$\frac{\partial |a\rangle}{\partial z} = i\gamma\kappa(\langle b|b\rangle |a\rangle + |b\rangle \langle b|a\rangle), \tag{2a}$$

$$\frac{\partial |b\rangle}{\partial z} = i\gamma\kappa(\langle b|b\rangle |b\rangle), \tag{2b}$$

where the term $|b\rangle \langle b|a\rangle$ in (2a) accounts for the XMM. We dropped the four wave mixing product $|b\rangle \langle a|b\rangle$ and $|a\rangle \langle b|a\rangle$ since they are phase mismatched in our system.

Considering the simplest case, i.e., a two mode coupling system. We treat (2b) first by writting the pump light $|b\rangle$ as

$$|b\rangle = \sqrt{P_p} \left(p |\phi_1\rangle e^{i\zeta_{b1} z} + q |\phi_2\rangle e^{i\zeta_{b2} z} \right), \tag{3}$$

where P_p is the power of pump light and ζ_{b1} and ζ_{b2} account for the nonlinear propagation constant induced by the SPM of the pump light itself. $|\phi_1\rangle$ and $|\phi_2\rangle$ are the eigenstates of the optical waveguides without the pump-induced nonlinearity, satisfying orthonormality, i.e., $\langle \phi_i|\phi_j\rangle = \delta_{ij}\ (i, j = 1, 2)$. p, q are the normalized complex amplitude and satisfy $|p|^2 + |q|^2 = 1$. It is found that $\zeta_{b1} = \zeta_{b2} = \gamma\kappa P_p$ by inserting (3) in the (2b). It's worthy to point out that the mode profile of the pump light $|b\rangle$ does not change with transmission, apart from the accumulated phase.

To find the eigenmodes of the new system given by (2a), we can write the probe light $|a\rangle$ as

$$|a\rangle = \sqrt{P_s} \left(m |\phi_1\rangle + n |\phi_2\rangle \right) e^{i\zeta z}, \tag{4}$$

where P_s is the power of the probe light and the eigenvalue ζ accounts for the nonlinear propagation constant induced by the pump light. m, n are the normalized complex amplitude, satisfying $|m|^2 + |n|^2 = 1$. Inserting (3) and (4) into (2a) and considering the orthonormality between modes, we obtain

$$\zeta \left(m |\phi_1\rangle + n |\phi_2\rangle \right) = \gamma\kappa P_p \left[\left(|p|^2 + |q|^2 \right) \left(m |\phi_1\rangle + n |\phi_2\rangle \right) + \left(p^* m + q^* n \right) \left(p |\phi_1\rangle + q |\phi_2\rangle \right) \right]. \tag{5}$$

By multipling (5) with $\langle \phi_1|$ and $\langle \phi_2|$, respectively, we can obtain the eigenvalue equation of the nonlinear system as follows

$$H(m \quad n)^T = \zeta (m \quad n)^T$$

where matrix H represents the Hamiltonian of the new system, and is given by

$$H = \gamma\kappa P_p \begin{pmatrix} |p|^2 + 1 & pq^* \\ qp^* & |q|^2 + 1 \end{pmatrix}.$$

The eigenvectors and eigenvalues are found to be

$$(m, n) = \{\{-q^*, p^*\}, \{p, q\}\}, \ \zeta = \gamma\kappa \{P_p, 2P_p\}. \tag{6}$$

Therefore, the evolution of the probe light over transmission can be described by the following analytical expression

$$|a\rangle = \sqrt{P_s} \sum_{i=1,2} a_i |\Phi_i\rangle e^{i\zeta_i L_{eff}},$$

where $|\Phi_1\rangle = -q^* |\phi_1\rangle + p^* |\phi_2\rangle$ and $|\Phi_2\rangle = p |\phi_1\rangle + q |\phi_2\rangle$ are the eigenmodes of the new system according to the eigenvectors. The nonlinear propagation constant ζ_i refers to the eigenvalue of corresponding eigenmodes. The modal coefficient $a_i = \langle \Phi_i|a\rangle$, satisfying $\sum_i |a_i|^2$=1 (i=1,2). The impact of transmission loss of the pump light can be taken into consideration by defining an effective length, i.e. $L_{eff} = (1 - e^{-\alpha L})/\alpha$, where α is the loss coefficient of the pump light. Note that $|\Phi_2\rangle$ is exactly the same pump mode, which means that the pump mode is always contained in the eigenmodes of the nonlinear system. Meanwhile, the eigenvalue of the pump mode is twice as large as the other eigenmode.

III. Simulations Result

We proceed to discuss how to apply the above formalism to examine the mode rotation indicated by Fig. 1(d), i.e., a system where two degenerate spatial modes with the same polarization is considered. In Fig. 1(d), we consider a two-mode system with $|\phi_1\rangle$/$|\phi_2\rangle$, referring to the normalized field of $LP_{11}^{a,x}$/$LP_{11}^{b,x}$ mode, respectively. According to Fig. 1(d), $|a\rangle = |\phi_1\rangle$ and $|b\rangle = \frac{1}{\sqrt{2}}(|\phi_1\rangle + |\phi_2\rangle)$, i.e., $p = q = \frac{1}{\sqrt{2}}$, $m = 1$ and $n = 0$. The power of the pump/probe light is 28/0 dBm, respectively. In this work we consider a step-index multimode fiber with numerical aperture $NA = 0.205$. The nonlinear coefficient of the simulated multimode fiber is $n_2 = 2.6 \times 10^{-20}\ m^2 W^{-1}$ and the effective area of the fundamental mode is $A_{eff} \approx 43.7\ um^2$, corresponding to $\gamma \approx 2.41\ W^{-1} km^{-1}$, $\kappa \approx 0.76$. The attenuation coefficient

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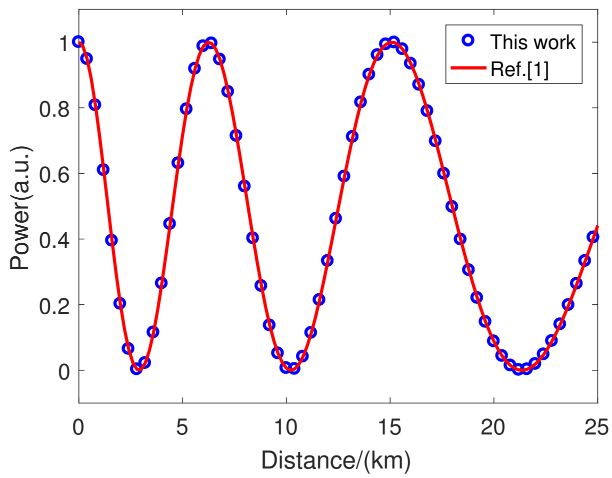

Fig. 2. Power distribution in the $LP_{11}^{a,x}$ mode at the probe light wavelength λ_a for a pump power of 28 dBm. The blue circles are the result of our work, and match perfectly well with the red solid line which is calculated according to [1].

of the fiber is $\alpha \approx 0.2\ dB/km$. According to (6), the eigenvectors and eigenvalues of the new system are found to be

$$(m,n) = \frac{1}{\sqrt{2}}\{\{-1,1\},\{1,1\}\},\ \zeta = \{1.156, 2.312\}\times 10^{-3} rad/m.$$

Therefore the eigenmodes of the nonlinear system are two LP_{11} modes with two lobes aligned at $45°$ and $-45°$. The predicted power distribution in the $LP_{11}^{a,x}$ mode at wavelength λ_a over transmission distance is shown in Fig. 2 by the blue circles. It's clear that due to the XMM-induced mode coupling, there is continuous exchange of energy between the $LP_{11}^{a,x}$ and the $LP_{11}^{b,x}$ mode. The extrema, corresponding to the case when the nonlinear phase shift accumulates to integer multiples of π, are the points where the probe light is is completely translated either to the $LP_{11}^{a,x}$ mode (maxima) or to the $LP_{11}^{b,x}$ mode (minima). The increase in the oscillating period of the curves is due to the transmission loss of the fiber. The results calculated according to [1] is shown by the red line in Fig. 2. This is done by calculating the value of $cos^2(\theta/2)$, where θ is the accumulated nonlinear phase shift defined according to Eq. (6) in [1]. It's clear that our method matches well with [1] which is further benchmarked against the coupled nonlinear Schrödinger equations (NLSE) [5].

In conclusion, we show that the Hamiltonian approach can apply pretty well for the XMM effect in a degenerate mode-group. By deriving the eigenmodes and eigenvalues of the nonlinear system, a simple analytical expression that describes the evolution of probe light can be obtained, and the results agree with the numerical simulations of the NLSE. That's to say, this kind of complicated vector nonlinear problem can be greatly reduced to a simple linear problem based on the coupled mode theory.

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