Hamiltonian formulation of cross-mode modulation without random mode mixing

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Abstract: General derivation of the Hamiltonian of cross-mode modulation system without random mode mixing is carried out and discussed under mode degenerancy. © 2019 The Author(s)

OCIS codes: (190.4370) Nonlinear optics, fibers; (060.2330) Fiber optics communications; (060.4370) Nonlinear optics, fibers.

1. Introduction

The cross-mode modulation (XMM) in multimode optical waveguides induced by third order non-linearity has received broad interests recently due to the rapid progresses in spatial division multiplexing technologies developed for optical fiber communications. Multimode nonlinear Schrodinger equation (MNLSE) [1] are widely used to describe the dispersion and nonlinear effects in multimode optical waveguides, including XMM effect [2], which is routinely modelled numerically due to the coexistence of dispersion and nonlinearity. Fortunately, in a commonly applied pump-probe configuration where the evolution of a weak probe light is determined by a strong pump light, the nonlinear problem of XMM can be effectively linearized, rendering a completely different yet powerful treatment, i.e., the Hamiltonian approach. In an initial attempt of Hamiltonian approach in analyzing the XMM in degenerate mode group with random mode mixing (RMM), the prediction as well as manipulation of the probe mode distribution are revealed with a clear picture of nonlinear birefringence [3]. Notably, our previous work [3] is typically for optical fibers, where RMM plays a relevant role. In integrated nonlinear photonic waveguides or other broad scenarios [4, 5], where RMM is negligible, such a Hamiltonian approach of linearizing the complex nonlinear dynamics is also very appearing. In this work, we propose a generalized Hamiltonian approach that is capable of analyzing the XMM without RMM and discussed under mode degenerancy.

2. Theoretical framework

We start from MNLSE for interaction between mode and neglect loss and dispersion [1], $\frac{\partial \vec{E}}{\partial z} = i\gamma \sum_{jhkm} C_{jhkm} E_h^* E_k E_m \hat{e}_j$, where $\vec{E}(z,t)$ represents the optical field envelope of $2N$ spatial modes including polarization. And $\hat{e}_j$ represents the orthogonal mode basis. C_{jhkm} account for the spatial mode overlap integrals, and γ is the nonlinear parameter. Loss can be included by defining an effective length [3]. We consider the pump-probe configuration in a two modes coupling system, where the two modes may be different from spatial distribution, polarization, or both. The total optical field at the input of the nonlinear waveguide can be written as $\vec{E} = \vec{a} + \vec{b}$, where $\vec{a} = a_1 e^{i\beta_1 z}\hat{e}_1 + a_2 e^{i\beta_2 z}\hat{e}_2$ and $\vec{b} = b_1 e^{i\beta_1 z}\hat{e}_3 + b_2 e^{i\beta_2 z}\hat{e}_4$ with modal label $j = 1,2$ corresponding to the two modes at λ_a (probe light) while $j = 3,4$ to the same modes but at wavelength λ_b (pump light). β_1 and β_2 represent the linear propagation constants of the two modes and $\Delta\beta = \beta_1 - \beta_2$. The coupled mode equations can be cast as:

$$\frac{\partial}{\partial z}\begin{pmatrix} a_1 & a_2 \end{pmatrix}^T = iH_a \begin{pmatrix} a_1 & a_2 \end{pmatrix}^T, \tag{1a}$$

$$\frac{\partial}{\partial z}\begin{pmatrix} b_1 & b_2 \end{pmatrix}^T = iH_b \begin{pmatrix} b_1 & b_2 \end{pmatrix}^T, \tag{1b}$$

where $H_a = \begin{pmatrix} H_{a_{11}} & H_{a_{12}} \\ H_{a_{21}} & H_{a_{22}} \end{pmatrix}, H_b = \begin{pmatrix} H_{b_{11}} & H_{b_{12}} \\ H_{b_{21}} & H_{b_{22}} \end{pmatrix}$,

$$H_{a_{11}} = \gamma\{(C_{1313}+C_{1331})|b_1|^2 + (C_{1414}+C_{1441})|b_2|^2 + (C_{1314}+C_{1341})e^{-i\Delta\beta z}b_1^* b_2 + (C_{1413}+C_{1431})e^{i\Delta\beta z}b_1 b_2^*\},$$

$$H_{a_{12}} = \gamma\{(C_{1323}+C_{1332})e^{-i\Delta\beta z}|b_1|^2 + (C_{1424}+C_{1442})e^{-i\Delta\beta z}|b_2|^2 + (C_{1423}+C_{1432})b_1 b_2^* + (C_{1342}+C_{1324})e^{-2i\Delta\beta z}b_1^* b_2\},$$

$$H_{a_{21}} = \gamma\{(C_{2313}+C_{2331})e^{i\Delta\beta z}|b_1|^2 + (C_{2414}+C_{2441})e^{i\Delta\beta z}|b_2|^2 + (C_{2314}+C_{2341})b_1^* b_2 + (C_{2431}+C_{2413})e^{2i\Delta\beta z}b_1 b_2^*\},$$

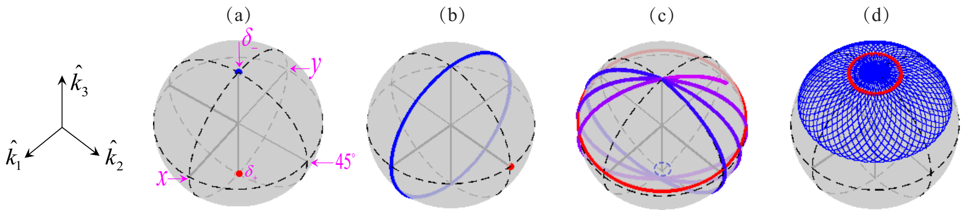

Fig. 1. Evolution of the SOP of pump light (red point/lines) and probe light (blue point/lines) on the *Poincaré sphere* under different initial pump SOP: (a) δ_+, (b) 45° linear polarization, elliptical polarization with ellipticity angles of (c) 3° and (d) 36°.

$$H_{a_{22}} = \gamma\{(C_{2424}+C_{2442})|b_2|^2+(C_{2323}+C_{2332})|b_1|^2+(C_{2324}+C_{2342})e^{-i\Delta\beta z}b_1^*b_2+(C_{2432}+C_{2423})e^{i\Delta\beta z}b_1b_2^*\},$$

and

$$\begin{aligned}
H_{b_{11}} &= \gamma\{C_{3333}|b_1|^2+(C_{3434}+C_{3443})|b_2|^2+C_{3433}e^{i\Delta\beta z}b_1b_2^*\},\\
H_{b_{12}} &= \gamma e^{-i\Delta\beta z}\{C_{3344}e^{-i\Delta\beta z}b_1^*b_2+(C_{3334}+C_{3343})|b_1|^2+C_{3444}|b_2|^2\},\\
H_{b_{21}} &= \gamma e^{i\Delta\beta z}\{C_{4433}e^{i\Delta\beta z}b_1b_2^*+(C_{4443}+C_{4434})|b_2|^2+C_{4333}|b_1|^2\},\\
H_{b_{22}} &= \gamma\{C_{4444}|b_2|^2+(C_{4343}+C_{4334})|b_1|^2+C_{4344}e^{-i\Delta\beta z}b_1^*b_2\}.
\end{aligned}$$

It is clear that the components of H_a is independent of a_1 and a_2, which means that Eq. (1a) can be solved as a standard eigenvalue problem. However, the evolution of the pump light is in general dependent on the mode decomposition of the pump light itself, i.e., b_1 and b_2, as indicated by H_b. In other words, this means that the evolution of the pump light can not be simplified using Hamiltonian approach. Since the development of the probe light is determined by the pump light, it is thus difficult to apply the Hamiltonian approach to the probe light as well. Fortunately, the spatial dependence of the Hamiltonian of the pump light can be eliminated when $\Delta\beta$ approaches zero. Considering the XMM coupling between the x and y components of the fundamental modes of an optical fiber without birefringence, i.e., $\Delta\beta = 0$. By neglecting the frequency dependence of the transverse mode profile, the nonzero coefficients of C_{jhkm} appear in Eq. (1) are $C_{1313} = C_{1331} = C_{2424} = C_{2442} = C_{3333} = C_{4444} = 1, C_{1441} = C_{1423} = C_{2314} = C_{2332} = C_{3443} = C_{4334} = 2/3, C_{1342} = C_{1324} = C_{2431} = C_{2413} = C_{3344} = C_{4433} = 1/3$. It turns out that H_b can be greatly simplified using circularly polarized components defined as $a_\pm = (a_1 \pm ia_2)/\sqrt{2}, b_\pm = (b_1 \pm ib_2)/\sqrt{2}$. The symbol $+$ and $-$ represent right- and left-handed circularly polarized states, denoted as δ_+ and δ_-, respectively. For convenience Dirac notations are adopted in the following derivations and Eq. (1) becomes $\frac{\partial|a\rangle}{\partial z} = iH'_a|a\rangle, \frac{\partial|b\rangle}{\partial z} = iH'_b|b\rangle$, where $|a\rangle = (a_+ \quad a_-)^T, |b\rangle = (b_+ \quad b_-)^T$, and by defining the initial condition of $|b\rangle$ at $z=0$ as $\sqrt{P_p}(p \quad q)^T$, with $|p|^2+|q|^2 = 1$, the Hamiltonian of the pump and probe can be written as

$$H'_b = \frac{2}{3}\gamma P_p\begin{pmatrix} |q|^2+1 & 0 \\ 0 & |p|^2+1 \end{pmatrix}, \qquad H'_a = \frac{4}{3}\gamma P_p\begin{pmatrix} 1 & pq^*e^{i\Delta\zeta_b z} \\ p^*qe^{-i\Delta\zeta_b z} & 1 \end{pmatrix}, \tag{2}$$

where $\Delta\zeta_b = \zeta_{b_1} - \zeta_{b_2}$ and the eigenvalues ζ_b is given by H'_b as $\zeta_b = \{\zeta_{b_1}, \zeta_{b_2}\} = \frac{2}{3}\gamma P_p\{|q|^2+1, |p|^2+1\}$.

3. Simulations result

So far an explicit expression of the probe Hamiltonian is written out, which is in general spatially dependent. In case of $pq = 0$ or $|p| = |q|$ ($\Delta\zeta_b = 0$), the spatial dependence can be eliminated. In the following, the evolution of the state of polarization (SOP) of the pump and probe light are elaborated and visualized on *Poincaré sphere* using Stokes vectors, as shown in Fig. 1. Pump is represented by red dots/lines and probe by blue/purple dots/lines. The stokes vectors of the pump and probe are defined as $\vec{m} = \langle b|\vec{\sigma}|b\rangle$and $\vec{n} = \langle a|\vec{\sigma}|a\rangle$, respectively, see details in [6]. In all cases, the initial SOP of the probe light is fixed at δ_-. The evolution pattern of the probe and pump light under different initial pump SOP are explained as follows. Figure 1(a) shows the case of $pq = 0$, where the initial SOP of the pump could be either at δ_+ or δ_- and δ_+ is used in this case. The corresponding eigenvectors and eigenvalues of the probe light are $\vec{u} = \{\{1,0\},\{0,1\}\}, \zeta_a = \frac{4}{3}\gamma P_p\{1,1\}$. The pump and probe light does not change over transmission since both the pump and probe are prepared in eigenstates. Actually, probe light launched in any polarization states does not change over transmission since the eigenvalues become degenerate in this case. Figure 1(b) shows the case of $|p| = |q|$, where the pump light is linearly polarized (LP). 45° LP is chosen in this case. It can be seen that the pump light does not evolve since that H'_b becomes proportional to an identity matrix, which is the reason why H'_a becomes z independent. The evolution of the probe light forms a single and closed circle on the *Poincaré sphere*, which is a general feature of z independent Hamiltonian. In the

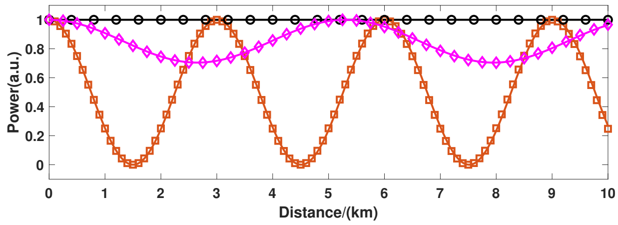

Fig. 2. Power distribution of the probe light in left-handed circularly polarized states (δ_-) for different initial pump states. Symbols and lines are calculated by Hamiltonian approach and MNLSE simulations, respectively. The numerical aperture of the simulated step-index multimode fiber is 0.205. The nonlinear coefficient of the fiber is $n_2 = 2.6\times10^{-20}m^2W^{-1}$, and the effective area of the fundamental mode is $A_{eff} \approx 42.3um^2$ corresponding to $\gamma \approx 2.49W^{-1}km^{-1}$.

most general situations, i.e., $pq \neq 0$ and $|p| \neq |q|$, H_a' is z dependent and the eigenvectors and eigenvalues are found to be $\vec{u} = \{\{\frac{|p||q|}{p^*q}e^{i\Delta\zeta_b z}, 1\}, \{-\frac{|p||q|}{p^*q}e^{i\Delta\zeta_b z}, 1\}\}, \zeta_a = \frac{4}{3}\gamma P_p\{1+|p||q|, 1-|p||q|\}$. Figure 1(c) shows the case when the initial SOP of the pump light is elliptically polarized with a small ellipticity angle of 3°. In contrast to previous cases, pump light evolves along transmission which leads to the z dependence of the probe Hamiltonian. In this case, pump light traces a circle on the *Poincaré sphere* very close to the equator. Due to the z dependence of the probe Hamiltonian, probe light walks around the sphere by starting from the north pole, passing through a point which is very close to the south pole and back to the north pole again, as shown by the blue and purple lines. Only the first few cycles are plotted in Fig. 1(c) for clearance and the color is gradually tuned from blue to purple to indicate the evolution. The SOPs of the probe light cover almost the full *Poincaré sphere* except a small area encircled by the dotted line around the south pole. Figure 1(d) illustrates the case when the ellipticity angle of the pump light increases to 36°. Similarly, pump light traces a circle parallel but further away from the equator while the probe state walks around the sphere, covering less than half of the *Poincaré sphere*.

To verify the proposed Hamiltonian approach, the power distribution of the probe light in δ_- over transmission distance is compared with that solved by the coupled MNLSE, as shown in Fig. 2. The parameters of the fiber used in simulations are given in the caption of Fig. 2. All curves start from 1 since the initial SOP of the probe light is set at δ_- in all cases. The probe evolutions under different pump conditions shown in Fig. 1(a), 1(b) and 1(d) are plotted with black circles, brown squares, magenta diamonds, respectively, corresponding to the situation where the probe light does not evolve, evolves with z independent Hamiltonian, and with z dependent Hamiltonian. The results calculated according to MNLSE [1] are shown by the solid lines of the corresponding color for comparison. It is clear that our method matches well with MNLSE simulations.

In conclusion, we focus on the linearization of XMM in the absence of RMM. We emphasis that the Hamiltonian approach contributes a general but simple picture to the understanding of XMM nonlinear problem. This work can be used as design rules for dynamic manipulation of the probe and pump modes based on XMM effect.

Funding. National Natural Science Foundation of China (NSFC) (61735006, 61775063); National Key Research and Development Program of China (2017YFA0305200); the Fundamental Research Funds for the Central Universities, HUST (2018KFYYXJJ055).

References

1. F. Poletti and P. Horak, "Description of ultrashort pulse propagation in multimode optical fibers," J. Opt. Soc. Am. B **25**, 1645–1654 (2008).
2. D. I. Kroushkov, G. Rademacher, and K. Petermann, "Cross mode modulation in multimode fibers," Opt. Lett. **38**, 1642-1644 (2013).
3. H. Yang, W. Chen, H. Hu, J. Xu, Y. Chen, and X. Zhang, "On the Hamiltonian form of cross-mode modulation in nonlinear optical waveguides," Opt. Lett. **43**, 5005-5008 (2018).
4. X. Wu and L. Tong, "Optical microfibers and nanofibers," Nanophotonics. **2**, 407-428 (2013).
5. N. M. Lpken, T. Hellwig, M. Schnack, J. P. Epping, K. J. Boller, and C. Fallnich, "Low-power broadband all-optical switching via intermodal cross-phase modulation in integrated optical waveguides," Opt. Lett. **43**, 1631-1634 (2018)
6. Qiang Lin and G. P. Agrawal, "Vector theory of cross-phase modulation: role of nonlinear polarization rotation," IEEE J. Quantum Electron. **40**, 958-964 (2004).