General Coupled Mode Theory in Non-Hermitian Waveguides

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Abstract— Coupled mode theory (CMT) can be traced back to 3 decades ago, and has been an indispensable tool of analyzing and designing waveguides, resonators, couplers or many other devices from microwave frequency to optical frequency [1], in both time and space domain. In the presence of loss and gain, the coupled mode equation on describing the mode hybridization of various waveguides or cavities, or cavities coupled to waveguides becomes intrinsically non-Hermitian. In such non-Hermitian waveguides, the conventional coupled mode theory (CCMT, see [1]) fails.

We generalize the coupled mode theory (GCMT) with a properly defined inner product based on reaction conservation. From reaction conservation, we provide a scheme of constructing general couple mode theory that can handle mode hybridization in non-Hermitian waveguides. Using a scalar inner product, we establish the equivalence between the self-adjointness of Maxwell's equations and reaction conservation. As for waveguide problems, the dimension of the self-adjoint relation need to be reduced from 3D to 2D, in which the formula turns out be non self-adjoint problem. Using coutering-propagating modes as the dual space of 2D non self-adjoint waveguide problem, the eigenmodes can be resolved from variational principles. Importantly, the 2D non self-adjoint relation can be elaborated into a set of coupled mode equation.

As an example, we study non-hermitian waveguides with balanced gain and losses using GCMT, CCMT as well as fullwave simulation (COMSOL), as shown in Figs. 1(a)–(d). In such so-called PT-symmetric waveguides, bifurcation occurs as the magnitude of the imaginary part of $\bar{\boldsymbol{\epsilon}}_r$ of two waveguides, i.e., $\bar{\boldsymbol{\epsilon}}_r = \bar{\boldsymbol{\epsilon}}_{r,0} + i\Delta\epsilon$ in core layer 1 and $\bar{\boldsymbol{\epsilon}}_r = \bar{\boldsymbol{\epsilon}}_{r,0} - i\Delta\epsilon$ in core layer 2, crosses a critical value as shown by the inset. The splitting in the dispersion diagram is confirmed by COMSOL where two different gap size between two core layers are considered, see Figs. 1(a)–(b). In contrast to excellent agreement between GCMT and fullwave simulation, CCMT fails to capture the major feature of PT-symmetric waveguide, as shown in Figs. 1(c)–(d). As for hermitian waveguide in Figs. 1(e), (f), one find that GCMT developed in this work gives the same results as CCMT does, agreeing well with full wave simulations.

In closing, the major difference of GCMT with CCMT is the special inner product that is selected to include optical materials that may contain losses and gain, and all the other aspects of constructing coupled mode thoery, i.e., variational principle, are essentially the same. CCMT has already widely accepted as an excellent method to study hermitian waveguides, and GCMT developed here can be taken as an complementary approach to study non-hermitian waveguide.

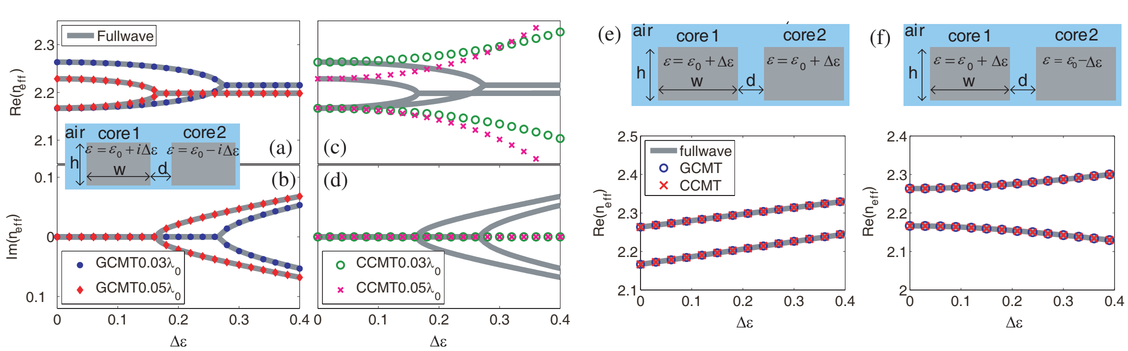

Figure 1: $\mathrm{Re}(n_{eff})$ and $\mathrm{Im}(n_{eff})$ versus $\Delta\epsilon$ using (a) (b) GCMT, and (c) (d) CCMT, and COMSOL (gray solid lines) with $h = 0.2\lambda_0$, $w = 0.3\lambda_0$, $d = 0.03\lambda_0$ or $0.05\lambda_0$, $\epsilon_0 = 10$. (e)–(f) $\mathrm{Re}(n_{eff})$ versus $\Delta\epsilon$ using GCMT (open circles), CCMT (red crosses) and COMSOL (gray solid lines) with $d = 0.03\lambda_0$.

REFERENCES

1. Haus, H. A., W. P. Huang, S. Kawakami, and N. A. Whitaker, *J. Lightw. Technol.*, Vol. 5, 16–23, 1987.
2. Xu, J. and Y. T. Chen, *Opt. Express*, Vol. 23, 11566–11575, 2015.