

FEM Modeling of Non-self-adjoint Waveguide Problems

Weijin Chen and Yuntian Chen

School of Optical and Electronic Information
Huazhong University of Science and Technology, Wuhan, China

Abstract— The traditional FEM can only calculate the reciprocal problem. Our new non-self-adjoint FEM can solve non-self-adjoint problem which contains inhomogeneous and anisotropic media where the tensor constitutive parameters have full nine components [1, 2]. In this contribution, we study a non-self-adjoint waveguide problem using finite element method. For a source free problem,

$$Lf = 0, \quad (1)$$

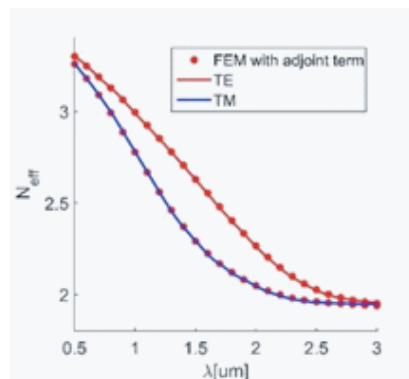
where L is differential operator and f is electromagnetic field. The original functional of this problem is showed in (2).

$$I = \langle f^a, Lf \rangle \quad (2)$$

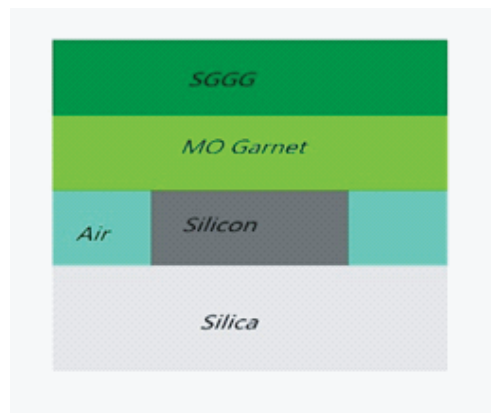
$$I^{non} = \langle f^a, Lf \rangle + \langle L^a f^a, f \rangle \quad (3)$$

As L is a non-self-adjoint operator, f can be explained to a set of primitive vector that is not complete, so we need another set of adjoint primitive vector which is called as trail function to supplement the solution space. Then the original and adjoint primitive vectors together form a complete solution space. So we add adjoint term in (3) where the permittivity and permeability tensors are adjoint of original ones.

To check out this method, we apply this method to a concrete problem which is discussed in Pintus's paper [3]. We get the weak form by deriving (3) and put it to the COMSOL. As shown in Figure 1(a), the result has a great agreement with accurate vectorial finite element method that is used by Pintus.



(a)



(b)

Figure 1: (a) Red point refer to the result of our method, red and blue line respectively refer to the N_{eff} of TE and TM which are from Accurate vectorial finite element method. (b) The structure of adjoint module, $n_{SGG} = 1.97$, $n_{Silica} = 1.46$, $n_{Silicon} = 3.45$.

REFERENCES

1. Chen, C. H. and C. D. Lien, "The variational principle for non-self-adjoint electromagnetic problems [J]," *IEEE Transactions on Microwave Theory & Techniques*, Vol. 28, No. 8, 878–886, 1980.
2. Xu, J. and Y. Chen, "General coupled mode theory in non-Hermitian waveguides [J]," *Optics Express*, Vol. 23, No. 17, 22619–22627, 2015.
3. Pintus, P., "Accurate vectorial finite element mode solver for magneto-optic and anisotropic waveguides [J]," *Optics Express*, Vol. 22, No. 13, 15737–15756, 2014.