Emergence of effective optical gauge field in biaxial anisotropic cavity

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***Abstract*—We investigate the emergence of effective optical gauge field in biaxial anisotropic Fabry-Pérot cavities due to the coupling between waveguide modes. The intrinsic material anisotropy can induce an effective gauge field, leading to the transition from Abelian to non-Abelian optical phenomena as frequency varies. Such transition is realized by tuning the frequency across band crossing point, accompanied by different wave packet dynamics due to the anisotropy induced effective gauge field.**

***Index Terms*—gauge field, anisotropic cavity, spin hall effect, zitterbewegung**

I. Introduction

The concept of effective optical gauge fields [1], [2] in anisotropic Fabry-Pérot (FP) cavities emerges from the coupling between differently polarized cavity modes induced by the anisotropy of the intracavity material [3]–[6]. In particular, a two-dimensional confined optical system exhibiting energy splitting between transverse electric and transverse magnetic polarized modes can be mapped onto a Rashba–Dresselhaus Hamiltonian [3]. The photonic analog of the Zitterbewegung has also been observed with the real-space propagation of polariton wave packets in planar semiconductor micro cavities [5], [6].

Here, we explore the coupling between two eigen modes with identical parity but different polarization in a biaxial anisotropic optical FP cavity, and propose an effective optical gauge field, which exhibits a transition from Abelian to non-Abelian forms at the frequency dimension. This transition corresponds to a shift from the optical spin Hall effect (SPE) to Zitterbewegung (ZB) of light. We that the wave packet dynamics induced by anisotropy can be analogized to semi-classical particles described by gauge fields, convinced from both the wave packet dynamics and the propagation trajectories.

II. The unique band crossing in biaxial FP cavity

We focuses on the planar waveguide modes in optical FP cavity filled with biaxial anisotropic dielectric. Compared to the unixial case, the birefringence term of the dielectric tensor gives extra X-Y splitting under normal incidence, and bring the emergence of the diabolical points (DPs, $\hbar\omega_0 = \hbar c k_0$) with non-zero in-plane wave vector k_{x0}. Figure. 1 demonstrate the iso-frequency surface at different energy. At DPs, the centers of the circular dispersion is shifted from the origin due to the existence of an effective optical gauge potential $\mathcal{A}_y$ along k_y. While away from DPs, the splitting induced by birefringence generates another effective gauge potential $\mathcal{A}_x$ along k_x. This unique band crossing recalls the scheme of gauge field optics with frequency-dependent gauge potential

$$\hat{H}^{\mathrm{eff}}\left|\psi\right\rangle = \left[\frac{1}{2}\left(\hat{\mathbf{p}} - \hat{\mathcal{A}}\right)\cdot\overline{\mathbf{m}}^{-1}\cdot\left(\hat{\mathbf{p}} - \hat{\mathcal{A}}\right) + V_0\right]\left|\psi\right\rangle = 0. \tag{1}$$

Here $|\psi\rangle = (\varepsilon_x f_{xm}, \varepsilon_y f_{yn})^T$ represents as a two-component wave function, and $\hat{H}^{\mathrm{eff}}$ denotes the effective Hamiltonian of a non-relativistic spin-1/2 particle traveling in $SU(2)$ non-Abelian gauge potentials, where $\hat{\mathbf{p}} = -i\hat{\sigma}_0\partial_i\mathbf{e}^i \quad (i = 1, 2)$ is the canonical momentum operator with σ_0 being the 2D identity matrix, $\overline{\mathbf{m}} = \mathrm{diag}\left(m_x^{\mathrm{eff}}, m_y^{\mathrm{eff}}\right)$ resembles an effective anisotropic mass, in particular, $\hat{\mathcal{A}} = \mathcal{A}_x\hat{\sigma}_1 + \mathcal{A}_y\hat{\sigma}_2$ ($\hat{\sigma}_a$ $(a = 1, 2, 3)$ are Pauli matrices) can be interpreted as emergent non-Abelian vector potentials, and V_0 is an additional Abelian scalar potential.

Analogous to real electromagnetic (EM) fields, the emergent $SU(2)$ gauge field generate a non-Abelian magnetic field along z-axis

$$\hat{\mathcal{B}} = \nabla\times\hat{\mathcal{A}} - i\hat{\mathcal{A}}\times\hat{\mathcal{A}}. \tag{2}$$

But different from classical EM field theory, the second term of $\hat{\mathcal{B}}$ emerges due to the noncommutativity nature of the non-Abelian gauge field potential.

III. Transition from SPE to ZB

The wave packet dynamics can accurately express the transition from the Abelian type to non-Abelian type with the frequency-dependent effective gauge potential. The Abelian magnetic field (the first term in Eq. (2)) vanishes in homogeneous media, while the non-Abelian part (the second term in Eq. (2)) persists due to the noncommutativity of $\mathcal{A}_\mu$ $(\mu = 1, 2)$.

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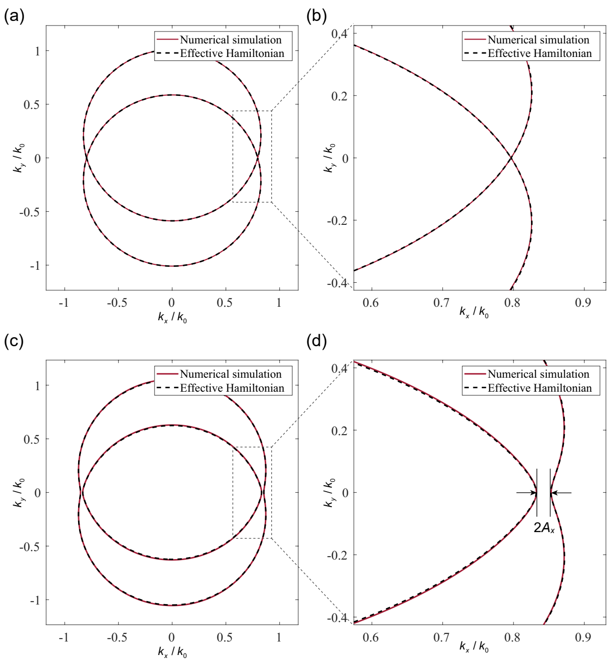

Fig. 1. The iso-frequency surface (a,b) at DPs, and (c,d) away from DPs. The emergent effective gauge potential $\mathcal{A}_x$ varies in frequency dimension.

The two-component wave function of light $|\psi\rangle$ behaves like a spin-1/2 spinor with pseudo-spin at a local point

$$\overrightarrow{s} = \langle\psi|\overrightarrow{\hat{\sigma}}|\psi\rangle/|\psi|^2. \quad (3)$$

At DPs, the shifted iso-frequency surface shows the decoupling of two eigen pseudo-spins ($\overrightarrow{s}_d = (0,1,0)$ and $\overrightarrow{s}_u = (0,-1,0)$. For normal incidence with $\overrightarrow{s}_0 = (-1,0,0)$, the two pseudo-spins will propagate dependently along different tangent directions of the iso-frequency surfaces. While away from DPs, the two eigen pseudo-spins are quasi-degenerate along k_x. And in the overlapped region, their superposed wave can be viewed as an intact "semiclassical particle" with an internal spin degree of freedom. The centroid trajectory of wave packet is

$$y\left(x\right) = Y_{ZB}\left[\sin\left(k_{ZB}\left(x - x_0\right) - \phi_0\right) + \sin\left(\phi_0\right)\right], \quad (4)$$

where x, y are the coordinates of centroid. The ZB amplitude and ZB wave number are

$$Y_{ZB} = \frac{-\mathcal{A}_2\sin\theta_0}{2k\mathcal{A}_1}, \quad k_{ZB} = \frac{2k\mathcal{A}_1}{k - \mathcal{A}_1\cos\theta_0}. \quad (5)$$

The centroid trajectory of the beam is determined by the initial spin $\overrightarrow{s}_0 = (\cos\theta_0, \sin\theta_0\cos\phi_0, \sin\theta_0\sin\phi_0)^\top$ at an angle θ_0 from $\hat{\sigma}_1$-axis. The ZB reaches the maximum when the initial spin $\overrightarrow{s}_0$ is perpendicular to the precession angular velocity $\overrightarrow{\Omega} = -\frac{2}{\mathbf{m}}\mathbf{k}\cdot\mathcal{A}_a\overrightarrow{\mathbf{e}}^a$. During the precession, the pseudo-spin component parallel to $\overrightarrow{\Omega}$ is conserved.

The time-averaged energy densities are presented in Figs. 2(a) and 2(b), respectively. The former exhibits polarization-dependent beam splitting, while the latter demonstrates a transverse tremor along the beam. As shown in Figs. 2(c) and 2(d), the centroid trajectory extracted from the full-wave results agree perfectly with the analytical expression.

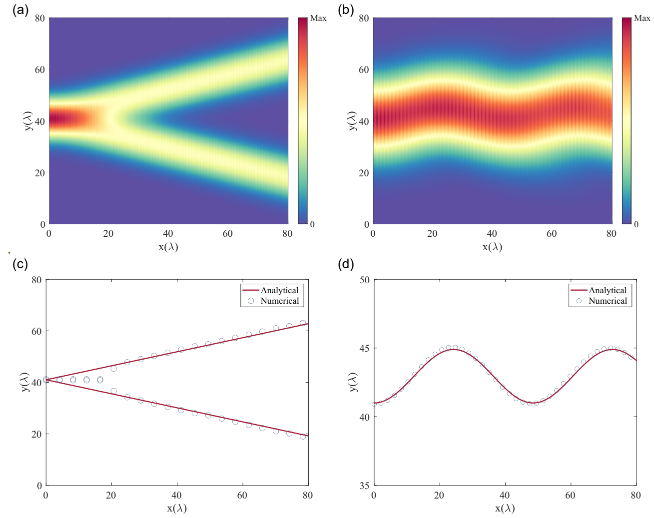

Fig. 2. The beam is emitted along x-direction with a Gaussian envelope in y-direction. The time-averaged energy densities of (a) SPE and (b) ZB. The dielectric tensor $\bar{\varepsilon}_r = \mathrm{diag}\left(1.96, 2.25, 3.61\right)$, and the cavity length $L = 500nm$. (c) and (d) give the comparison of centroid trajectories between analytical calculation and numerical simulation.

IV. Conclusion

We have demonstrated that non-trival gauge field can be engineered through the coupling between two waveguide modes with identical parity but different polarization within a biaxial anisotropic optical FP cavity. The unique band crossing and the resulting wave packet dynamics with frequency-dependent gauge fields are validated by both theoretical calculation and numerical simulation.

Acknowledgment

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