

Selective mode excitations for multi-mode scatterers through incident polarizations

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ABSTRACT

We obtain selective mode excitations for reciprocal multi-mode scatterers excited by incident plane waves. In the multi-mode regime, the excitation efficiencies for quasi-normal modes (QNMs) supported can be directly calculated in the far field through the incident directions and polarizations. The excitation of any QNM is optimized (eliminated) with incident polarization matching (orthogonal to) its radiation polarization opposite to the incident direction. As a result, for any pair of modes supported: when their radiation polarizations along the opposite incident direction are distinct, either of them can be selectively excited with the other fully suppressed; when their radiation polarizations are the same, both can only be simultaneously excited or suppressed; when the mode radiation along some direction is zero with polarization undefined, it cannot be excited with waves incident along the opposite direction, irrespective of the incident polarizations. An outstanding implication is that a multi-mode scatterer would become invisible when opposite to the incident direction all QNM radiation polarizations are identical (or some are undefined) and at the same time orthogonal to the incident polarization. We have further confirmed our theoretical analysis through numerical simulations, covering individual and coupled scatterers supporting degenerate and non-degenerate QNMs. The principles we have revealed are secured by the fundamental principle of reciprocity, and thus can be applied to not only general non-Hermitian electromagnetic systems with free-space radiations, but also other branches of wave physics where selective mode excitations are also essential.

1. Introduction

Mode analysis has been a fundamental tool for studies of almost all sorts of photonic structures, based on which many exceptional functionalities can be characterized and designed [1–4]. For open structures with free-space radiations (such as resonant cavities and finite scatterers [5,6]), the modes become quasi-normal, featuring finite Q-factors and complex eigenfrequencies [4,7]. Besides the modes themselves which are intrinsic, the photonic performance is also dependent on properties of the extrinsic exciting sources that decide the excitation efficiency and phase of each mode, since the interference among the excited modes and the source dictate the whole light-matter interaction process.

Photonic structures are generally multi-mode and thus external sources would simultaneously excite extra unwanted modes that would deteriorate the desired optical functionalities. For both closed waveguiding systems and open optical cavities, selective excitation and/or

suppression of targeted modes is of crucial significance, in particular for lasing, optical transmission and other related applications [8–12]. Except for some special scenarios where the principles of symmetries can be directly applied [1,4,13–15], existing approaches for selective mode excitations are based on various techniques, including but not limited to structured incident illuminations, sophisticated nonlinear mode dynamics and extra symmetries [16–24]. Conventional methods to calculate the excitation efficiency of each mode rely on cumbersome near-field volume integration measuring the matching of incident field and near field of the mode [1–4]. Those methods, though technically powerful and widely applied, usually overshadow rather than unveil the underlying physical picture and are more explanative than predictive, from which general and direct principles can be barely extracted. As a result, selective enhancement or suppression of mode excitations has long been a crucial while rather challenging task in optics and photonics.

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We have recently discovered that for the special case of a single-mode reciprocal scatterer excited by incident plane waves, the mode excitation efficiency can be directly calculated in the far field, as guaranteed by electromagnetic reciprocity [25,26]. In this work, we extend the investigations from single-mode to multi-mode scatterers [27], and reveal how our far-field perspective can rapidly decide which modes can be selectively excited and which can be selectively suppressed. It is shown for any pair of QNMs, along the direction opposite to the incident direction: (i) when the mode radiation polarizations are distinct, either of the QNMs can be selectively excited without exciting the other, as long as the incident polarization is tuned to be orthogonal to that of the latter; (ii) when the mode radiation polarizations are identical, the excitation efficiencies of both modes can only be simultaneously enhanced or suppressed; (iii) when a QNM radiation is zero and thus the polarization state is singular and undefined, this mode cannot be excited, whatever the incident polarization is. An outstanding implication of the above principles is that a multi-mode scatterer would become fully invisible when all QNM radiation polarizations along the opposite incident direction are identical (including undefined polarizations) and orthogonal to the incident polarization. All those points have been further verified by numerical calculations of individual and coupled scatterers, which support degenerate and non-degenerate QNMs. The principles we have exemplified with simple scatterers are generally applicable to open resonators and any non-Hermitian optical systems with free-space radiation, as long as the structures are reciprocal.

2. Theoretical analysis of QNM excitation efficiency in the far field

Throughout this paper, we confine our investigations to reciprocal multi-mode scatterers excited by incident plane waves [electric vector field $\mathbf{E}_i(\hat{\mathbf{r}}_i)$] of angular frequency ω , wavelength λ , and unit incident direction vector $\hat{\mathbf{r}}_i$. In the spectral regime of interest, the scatterer supports a set of discrete QNMs, with the m^{th} QNM characterized by eigenfield $\tilde{\mathbf{E}}_m(\mathbf{r})$ and complex eigenfrequency $\tilde{\omega}_m$ [7]. The total electric field excited can be expanded into the QNMs as:

$$\mathbf{E}(\mathbf{r}) = \sum \alpha_m \tilde{\mathbf{E}}_m(\mathbf{r}), \quad (1)$$

where α_m is the complex excitation (coupling) coefficient for the m^{th} QNM ($|\alpha_m|^2$ is corresponding to the excitation efficiency). In the far field, $\tilde{\mathbf{E}}_m(\mathbf{r})$ becomes transverse [denoted as $\tilde{\mathbf{E}}_m(\hat{\mathbf{r}})$, with $\hat{\mathbf{r}}$ being the unit radiation direction vector $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$], and thus both incident and QNM radiation far fields can be expressed as:

$$\mathbf{E}_i(\hat{\mathbf{r}}_i) = E_i(\hat{\mathbf{r}}_i) \mathbf{J}_i(\hat{\mathbf{r}}_i), \quad \tilde{\mathbf{E}}_m(\hat{\mathbf{r}}) = \tilde{E}_m(\hat{\mathbf{r}}) \tilde{\mathbf{J}}_m(\hat{\mathbf{r}}), \quad (2)$$

where $E_i(\hat{\mathbf{r}}_i)$ and $\tilde{E}_m(\hat{\mathbf{r}})$ are the corresponding scalar field amplitudes, for incident and QNM radiation fields, respectively; $\mathbf{J}_i(\hat{\mathbf{r}}_i)$ and $\tilde{\mathbf{J}}_m(\hat{\mathbf{r}})$ are the corresponding unit Jones (row) vectors [3]. For multi-mode scatterers, the excitation coefficient of each QNM can be calculated in the far field (without near-field volume integrations) according to the following relation secured by electromagnetic reciprocity [25,26]:

$$\alpha_m \propto \mathbf{J}_i(\hat{\mathbf{r}}_i) \tilde{\mathbf{J}}_m^\dagger(\hat{\mathbf{r}} = -\hat{\mathbf{r}}_i) E_i(\hat{\mathbf{r}}_i) \tilde{E}_m(\hat{\mathbf{r}} = -\hat{\mathbf{r}}_i), \quad (3)$$

where $\dagger$ denotes joint operations of complex conjugate and transpose; the restriction that $\hat{\mathbf{r}} = -\hat{\mathbf{r}}_i$ indicates that the excitation coefficient is correlated to the QNM radiation along the direction opposite to the incident direction, as imposed by the electromagnetic reciprocity [25, 26,28].

For a fixed incident direction, according to Eq. (3), along the opposite incident direction: (i) if the QNM radiation polarization fully matches that of the incident wave: $|\mathbf{J}_i(\hat{\mathbf{r}}_i) \tilde{\mathbf{J}}_m^\dagger(\hat{\mathbf{r}} = -\hat{\mathbf{r}}_i)| = 1$, the excitation is maximal; (ii) if the QNM radiation polarization is orthogonal to that of the incident wave $|\mathbf{J}_i(\hat{\mathbf{r}}_i) \tilde{\mathbf{J}}_m^\dagger(\hat{\mathbf{r}} = -\hat{\mathbf{r}}_i)| = 0$, the QNM would not be excited; (iii) If the QNM radiation is zero with the polarization state undefined, the QNM would not be excited irrespective of the incident polarizations. As a result, for any pair of modes along an arbitrarily

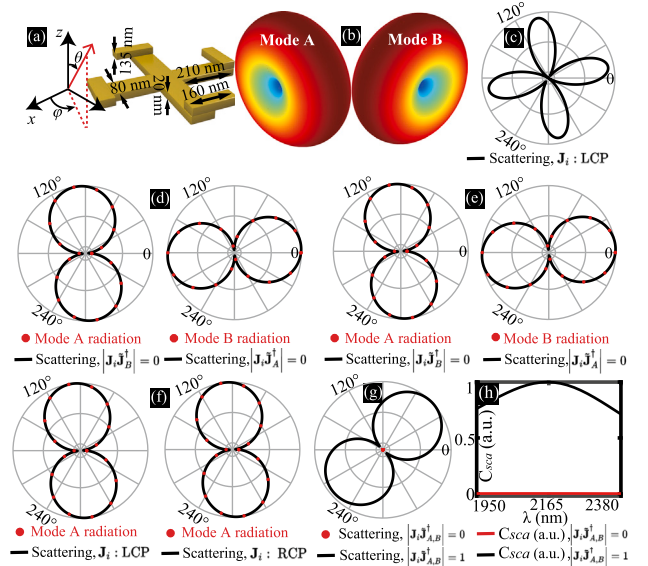


Fig. 1. (a) A gold structure that exhibits four-fold rotation symmetry, with all geometric parameters specified. The Cartesian coordinate system is also parameterized by azimuthal angle ϕ and polar angle θ . (b) Far-field radiation patterns (in terms of radiation intensity) for the two supported degenerate QNMs denoted by modes A and B. (c) Scattering pattern (in terms of angular scattering intensity) on the x-y plane for LCP wave incident along +z direction. Scattering and mode radiation patterns on the x-y plane for waves incident along +z direction [shown in (d)] and another direction of $\phi = 0^\circ$ and $\theta = 20^\circ$ [shown in (e)]. In both (d) and (e), both scenarios with different polarizations are shown (incident polarizations are orthogonal respectively to that of modes A and B radiation polarizations opposite to the incident direction). (f) Scattering and mode radiation patterns on the x-y plane and (h) scattering cross section spectra for the waves incident along the direction of $\phi = 45^\circ$ and $\theta = 90^\circ$. For (g) and (h) both scenarios of matched and orthogonal incident polarizations are shown. (d) $\tilde{\mathbf{J}}_A = [0.115, 0.955 + 0.2726i]$ and $\tilde{\mathbf{J}}_B = [0.995, -0.0793 + 0.0585i]$; (e) $\tilde{\mathbf{J}}_A = [0.1076, 0.958 + 0.265i]$ and $\tilde{\mathbf{J}}_B = [0.993, -0.114 + 0.034i]$; (f) $\tilde{\mathbf{J}}_A = [0.0018, 0.2159 + 0.9764i]$, and $\tilde{\mathbf{J}}_B$ is singular and undefined; (g) & (h) $\tilde{\mathbf{J}}_A = \tilde{\mathbf{J}}_B = [0.04, -0.0359 - 0.9985i]$. In (c)–(g), the incident wavelength is fixed at $\lambda = 2.07 \mu\text{m}$.

chosen direction, as long as their radiation polarizations are distinct, one of them can be selectively excited without exciting the other one for incident plane waves along the opposite direction. For an incident direction opposite to which the QNM radiation polarizations are identical, the excitations of both QNMs are simultaneously enhanced or suppressed. It is unlikely that two distinct modes along all directions exhibit the same radiation polarization, and thus selective excitation is generally accessible for at least some incident directions through tuning the incident polarization to be orthogonal to that of the undesired QNM. If there exists one direction ($\hat{\mathbf{r}} = \hat{\mathbf{r}}_0$) along which all QNMs have the same radiation polarization $\tilde{\mathbf{J}}_m = \tilde{\mathbf{J}}_0$, for incident wave along the opposite direction ($\hat{\mathbf{r}}_i = -\hat{\mathbf{r}}_0$) and orthogonal polarization ($|\mathbf{J}_i \tilde{\mathbf{J}}_0^\dagger| = 0$), none of the QNM would be excited and thus the scatterer would become fully invisible.

3. Numerical results for multi-mode reciprocal scatterers

To further exemplify the theoretical principles listed in the last section, we turn to specific scatterers for verifications. Without loss of generality, the scatterers consist of gold with effective permittivity curve fitted from measured data in Ref. [29], and all numerical results presented in this study are obtained using COMSOL Multiphysics.

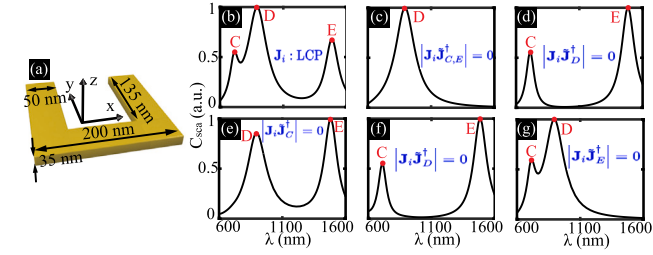


Fig. 2. (a) A gold SRR with all geometric parameters specified. (b) Scattering cross section spectra for LCP waves incident along +z direction. Scattering cross section spectra for incident (along +z direction) polarizations orthogonal to that of mode C & E [shown in (c)] and orthogonal to that of mode D [shown in (d)]. (e)–(g) Scattering cross section spectra for incident (along the direction of $\varphi = 180^\circ$ and $\theta = 60^\circ$) polarizations orthogonal to that of mode C, D and E, respectively. In (b)–(g) the positions of excited QNMs C–E are marked. (b)–(d) $\vec{J}_C = [1, 0]$, $\vec{J}_D = [0, 1]$ and $\vec{J}_E = [1, 0]$. (e)–(g) $\vec{J}_C = [0.8147, -0.0174 + 0.5796i]$, $\vec{J}_D = [0.0022, -0.4472 - 0.8944i]$ and $\vec{J}_E = [0.0864, -0.06 - 0.9944i]$.

3.1. Scatterers that support degenerate QNMs

We start with a structure schematically shown in Fig. 1(a) (together with a parameterized coordinate system with azimuthal angle φ and polar angle θ), where all the geometric parameters are also specified. This structure manifests four-fold rotation symmetry, which secures a pair of degenerate QNMs (denoted by modes A and B) with identical complex eigenfrequency $\tilde{\omega}_{A,B} = 9.1 \times 10^{14} - 5.53 \times 10^{12}i$ [rad/s], and their far-field radiation patterns (in terms of radiation intensity) are shown in Fig. 1(b). The two patterns can be transformed to each by a $\pi/2$ rotation along z-axis, being consistent with the rotation symmetry of the scatterer.

Firstly we shine light along the +z direction and show the effect of selective mode excitation through polarization tuning [the radiation polarizations of modes A and B along the opposite incident direction (-z) are distinct; in this paper details of polarizations are specified in the figure captions]. For incident left-handed circularly-polarized (LCP) light [with the angular frequency fixed as the resonant QNM frequency $\omega = \text{Re}(\tilde{\omega}_{A,B})$ ($\lambda = 2.07 \mu\text{m}$)], the scattering pattern (in terms of angular scattering intensity) on the x-y plane is shown in Fig. 1(c), where both modes A and B are excited (four lobes of the scattering patterns are observed). We then tune the incident polarization to be orthogonal to that of mode A ($|\vec{J}_A \cdot \vec{J}_A^\dagger| = 0$) and B ($|\vec{J}_B \cdot \vec{J}_B^\dagger| = 0$) along -z direction, respectively. The corresponding scattering patterns and mode radiation patterns are shown in Fig. 1(d), the agreement between which confirms that only one mode is excited with the other fully suppressed (only two pattern lobes are observed for each scenario). We also demonstrate the same effect in Fig. 1(e) for another incident direction ($\varphi = 0^\circ$ and $\theta = 20^\circ$) opposite to which the mode radiation polarizations are also different.

As is shown in Figs. 1(d)–1(e), there are four directions on the x-y plane along which either mode A or B does not radiate. We choose one such direction ($\varphi = 94.5^\circ$ and $\theta = 90^\circ$; along this direction mode B does not radiate) and shine incident wave along the opposite direction. According to our analysis in the last section, mode B would not be excited, whatever the incident polarization is. This is confirmed by the scattering patterns (together with radiation patterns of mode A) shown in Fig. 1(f), for LCP and RCP (right-handed circularly-polarized) incident waves, respectively.

We have further identified a direction ($\varphi = 45^\circ$ and $\theta = 90^\circ$) along which the radiation polarizations of modes A and B are identical. For waves incident opposite to this identified direction, excitations of modes A and B are simultaneously enhanced (optimized with $|\vec{J}_{A,B} \cdot \vec{J}_{A,B}^\dagger| = 1$) or suppressed (fully eliminated with $|\vec{J}_{A,B} \cdot \vec{J}_{A,B}^\dagger| = 0$; the scatterer becomes

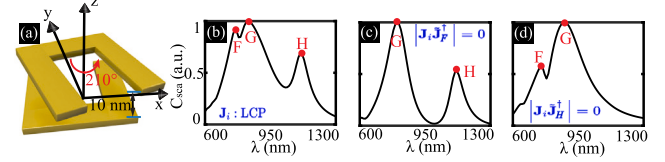


Fig. 3. (a) A pair of coupled SRRs (twisted by 210° and displaced by 10 nm with respect to each other) with geometric parameters specified. (b) Scattering cross section spectra for LCP wave incident along +z direction. (c)–(d) Scattering cross section spectra for incident (along +z direction) polarizations orthogonal to that of mode F and H, respectively. In (b)–(d) the positions of excited QNMs F, G and H are marked, and $\vec{J}_F = [0.3272, -0.7967 - 0.51i]$, $\vec{J}_G = [0.3658, 0.92 - 0.1335i]$ and $\vec{J}_H = [0.03, -0.986 - 0.1643i]$.

invisible). This is manifest from both the scattering pattern on the x-y plane shown in Fig. 1(g), and the scattering cross section spectra shown in Fig. 1(h).

3.2. Individual scatterers that support non-degenerate QNMs

We now proceed to study the seminal split-ring resonators (SRRs) that have played a foundational role for the field of metamaterials and more generally nanophotonics. It is well known that the SRR supports both electric and magnetic modes, and different incident directions and polarizations would excite different modes [30,31]. Previous interpretations of such excitation selectivity rely on a near-field approach that focuses on the local charge-current distributions of the modes and their matching with the incident field, which works only for simple structures and lower-order modes. For sophisticated structures or higher-order modes for which the near-field charge-current distributions become rather complex, the near-field perspective can barely predict which modes would be excited and which would not. In contrast, our far-field approach outlined in the last section is not constrained by the structure complexity or mode order, and Eq. (3) can be directly applied for any reciprocal scatterers and QNMs of any order.

A gold SRR is shown schematically in Fig. 2(a) and its normalized scattering cross section spectra for LCP wave incident along +z direction are shown in Fig. 2(b). For this spectral regime of interest, three QNMs (modes C, D and E) are marked with their corresponding complex eigenfrequencies being: $\tilde{\omega}_C = 2.9 \times 10^{15} - 2.06 \times 10^{14}i$ [rad/s], $\tilde{\omega}_D = 2.18 \times 10^{15} - 2.7 \times 10^{14}i$ [rad/s], and $\tilde{\omega}_E = 1.29 \times 10^{15} - 3.85 \times 10^{13}i$ [rad/s]. Along the opposite incident direction (-z), modes C and E have the same radiation polarization ($\vec{J}_C = \vec{J}_E$) and thus their excitations can only be simultaneously enhanced or suppressed (there is no selective excitation between modes C and E). We then show the scenarios of fully suppressing modes C & E ($|\vec{J}_{C,E} \cdot \vec{J}_{C,E}^\dagger| = 0$) and mode D ($|\vec{J}_D \cdot \vec{J}_D^\dagger| = 0$), through the spectra shown respectively in Figs. 2(c) and 2(d).

We have further identified a direction ($\varphi = 180^\circ$ and $\theta = 60^\circ$) along which the radiation polarizations of all modes are different, and thus selective excitations between any pair of modes are obtainable. The selective full suppressions of each mode are showcased in Figs. 2(e)–2(g), with waves incident opposite to this identified direction and incident polarizations orthogonal to that of the fully suppressed mode, respectively [mode C suppression in Fig. 2(e); mode D suppression in Fig. 2(f); mode E suppression in Fig. 2(g)].

3.3. Coupled scatterers that support non-degenerate QNMs

At the end, We turn to the more sophisticated configuration of a coupled SRR pair [twisted by 210° and displaced by 10 nm with respect to each other; each SRR is identical to the one in Fig. 2(a)] shown in Fig. 3(a). Its scattering cross section spectra (each normalized) with incident (along +z) LCP wave are shown in Fig. 3(b), and three excited QNMs (modes F, G and H) are marked, with their corresponding

complex eigenfrequencies being: $\tilde{\omega}_F = 2.47 \times 10^{15} - 2.7 \times 10^{14}i$ [rad/s], $\tilde{\omega}_G = 2.17 \times 10^{15} - 4.1 \times 10^{14}i$ [rad/s], and $\tilde{\omega}_H = 1.34 \times 10^{15} - 3.1 \times 10^{13}i$ [rad/s]. For incident polarizations orthogonal to the radiation polarization of mode F, mode F can be selectively fully suppressed, with the other two modes (G and H) excited, as is clearly shown in Fig. 3(c). While for a different incident polarization orthogonal to the radiation polarization of mode G, mode G can be selectively fully suppressed, with the other two modes (F and H) excited, as is clearly shown in Fig. 3(d).

4. Conclusions

In this work, we study selective mode excitations for reciprocal multi-mode scatterers excited by plane waves of varying polarizations. It is revealed that for any direction along which a pair of QNMs have different radiation polarizations, either of them can be selectively excited with the other fully suppressed. This is obtained by shining incident waves opposite to this direction with their polarization orthogonal to that of the fully suppressed mode. For directions along which two modes have the same radiation polarization, selective excitations between them are inaccessible: their excitation efficiencies are simultaneously enhanced or suppressed with varying incident polarizations. An extreme such scenario is that all QNMs supported have identical polarization along some direction, and then for a wave incident opposite to this direction with an orthogonal polarization, no QNMs would be excited and thus the scatterer would become fully invisible. For a QNM that does not radiation along some direction where the polarization is undefined, waves incident opposite to this direction would not excite this QNM, whatever their polarization is. Those principles revealed are not only applicable to scattering particles, but also to any reciprocal photonic systems that have free-space radiation, as secured by the electromagnetic reciprocity. Selective mode excitations play a crucial role not only for photonic systems [8–12], but also for systems that support modes of other forms (e.g. acoustic, seismic and matter wave modes) [32–35]. As a result, our results can potentially trigger similar explorations from the perspective of reciprocity principle in other branches of wave physics.

CRediT authorship contribution statement

ChaoFan Zhang: Methodology. **PengXiang Wang:** Writing – original draft, Software. **Yuntian Chen:** Writing – review & editing, Conceptualization. **Wei Liu:** Writing – review & editing, Visualization, Methodology, Conceptualization.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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